EE 330
Lecture 27

Small-Signal Analysis

- MOSFET Model Extensions
- Biasing (a precursor)

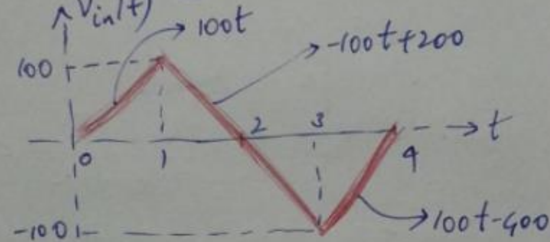
Two-Port Amplifier Modeling

Given:

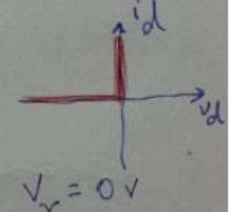
$V_{in}(t)$: triangular bipolar with

$V_{p-p} = 200V$, $T = 4$ sec

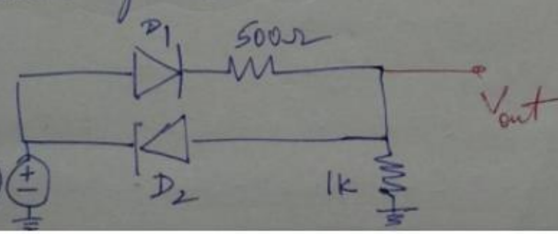
Consider one cycle of $V_{in}(t)$



Since nothing is mentioned about diodes we will assume them as ideal & its $i_d - v_d$ characteristics are as shown



Consider given circuit



Exam Schedule

Exam 1	Friday Sept 24
Exam 2	Friday Oct 22
Exam 3	Friday Nov 19
Final	Tues Dec 14 12:00 p.m.

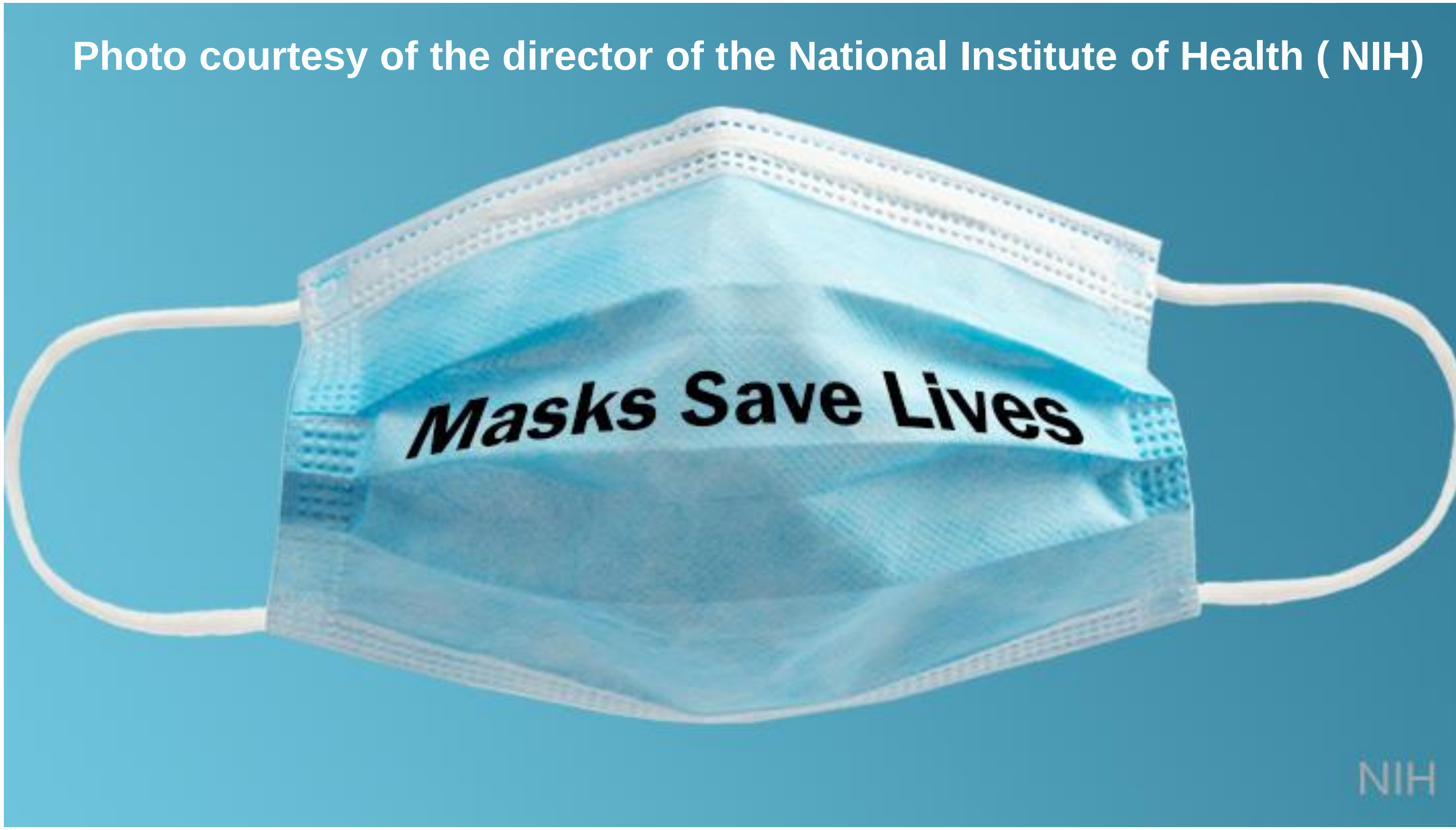
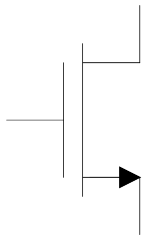


Photo courtesy of the director of the National Institute of Health (NIH)

As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

Small Signal Model of MOSFET

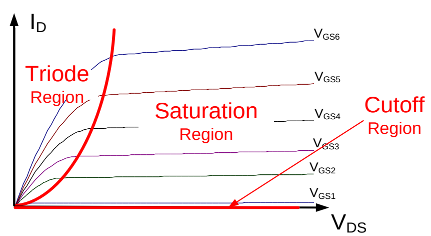


Large Signal Model

$$I_G = 0$$

3-terminal device

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$



MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region

Review from last lecture

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

Small-signal model:

$$y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} = 0 \qquad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q} = 0$$

$$y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} = 2\mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^1 \left.(1 + \lambda V_{DS})\right|_{\vec{V} = \vec{V}_Q} = \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)(1 + \lambda V_{DSQ})$$

$$y_{21} \cong \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

$$y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q} = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \left. \lambda \right|_{\vec{V} = \vec{V}_Q} \cong \lambda I_{DQ}$$

Small Signal Model of MOSFET

Saturation Region Summary

Nonlinear model:

$$\begin{cases} I_G = 0 \\ I_D = \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1+\lambda V_{DS}\right) \end{cases}$$

Small-signal model:

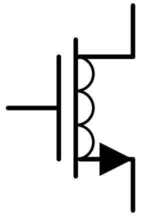
$$\begin{cases} i_G = y_{11} v_{GS} + y_{12} v_{DS} = 0 \\ i_D = y_{21} v_{GS} + y_{22} v_{DSE} \end{cases}$$

$$y_{11} = 0 \qquad y_{12} = 0$$

$$y_{21} = g_m \cong \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right) \qquad y_{22} = g_0 \cong \lambda I_{DQ}$$

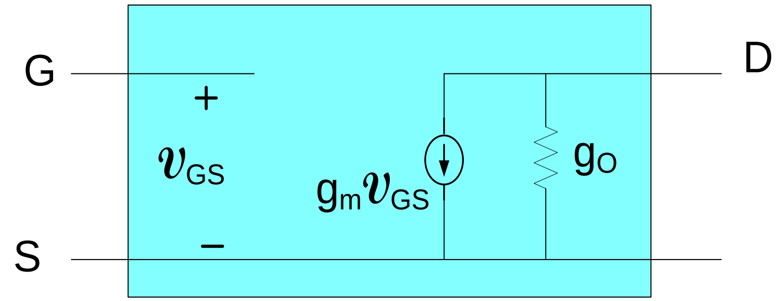
Review from last lecture

Small-Signal Model of MOSFET



$$g_m = \mu C_{OX}\frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o \cong \lambda I_{DQ}$$



Alternate equivalent expressions for g_m:

$$I_{DQ} = \mu C_{OX}\frac{W}{2L}\left(V_{GSQ} - V_T\right)^2\left(1+\lambda V_{DSQ}\right) \cong \mu C_{OX}\frac{W}{2L}\left(V_{GSQ} - V_T\right)^2$$

$$g_m = \mu C_{OX}\frac{W}{L}\left(V_{GSQ} - V_T\right)$$

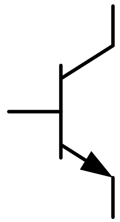
$$g_m = \sqrt{2\mu C_{OX}\frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

Review from last lecture

Small Signal Model of BJT

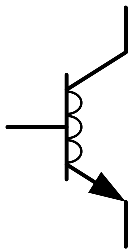
Nonlinear model:



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$

$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$y_{11} = g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{1}{V_t}\frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}\right|_{\vec{V}=\vec{V}_Q} = \frac{I_{BQ}}{V_t} \cong \frac{I_{CQ}}{\beta V_t}$$

$$y_{21} = g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{1}{V_t} J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1 + \frac{V_{CE}}{V_{AF}}\right)\right|_{\vec{V}=\vec{V}_Q} = \frac{I_{CQ}}{V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{22} = g_o = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{J_S A_E e^{\frac{V_{BE}}{V_t}}}{V_{AF}}\right|_{\vec{V}=\vec{V}_Q} \cong \frac{I_{CQ}}{V_{AF}}$$

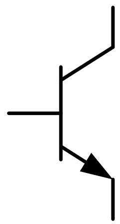
Note: usually prefer to express in terms of I_{CQ}

Review from last lecture

Small Signal Model of BJT

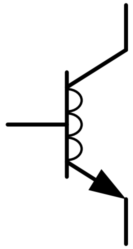
Forward Active Region Summary

Nonlinear model:



$$\begin{cases} I_B = \dfrac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \dfrac{V_{CE}}{V_{AF}}\right) \end{cases}$$

Small-signal model:



$$\begin{cases} i_B = y_{11} v_{BE} + y_{12} v_{CE} \\ i_C = y_{21} v_{BE} + y_{22} v_{CE} \end{cases}$$

$$y_{11} = g_\pi \cong \frac{I_{CQ}}{\beta V_t}$$

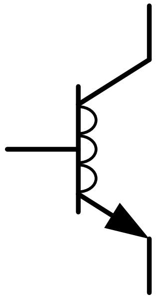
$$y_{21} = g_m = \frac{I_{CQ}}{V_t}$$

$$y_{12} = 0$$

$$y_{22} = g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Review from last lecture

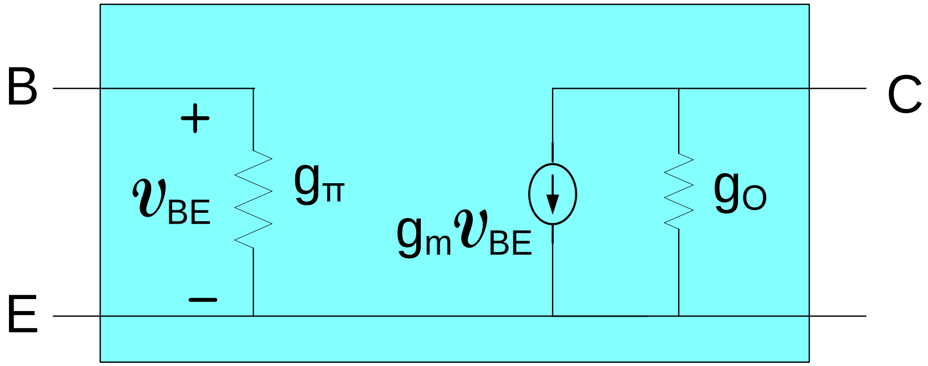
Small Signal Model of BJT



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$
$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$i_B = g_\pi v_{BE}$$
$$i_C = g_m v_{BE} + g_o v_{CE}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_m = \frac{I_{CQ}}{V_t} \qquad g_o = \frac{I_{CQ}}{V_{AF}}$$

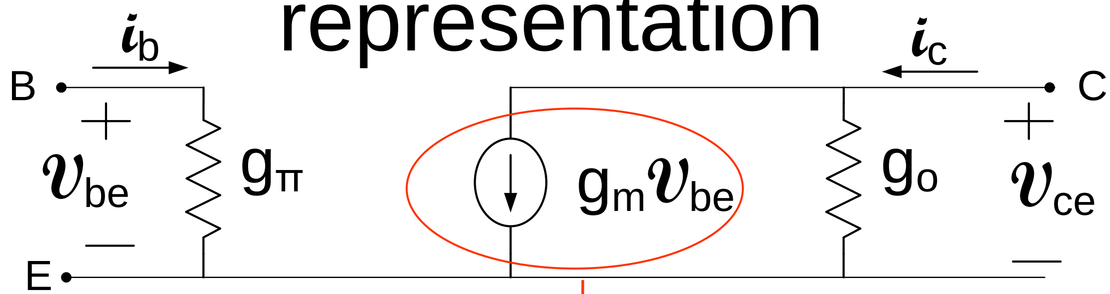


An equivalent circuit

y-parameter model using “g” parameter notation

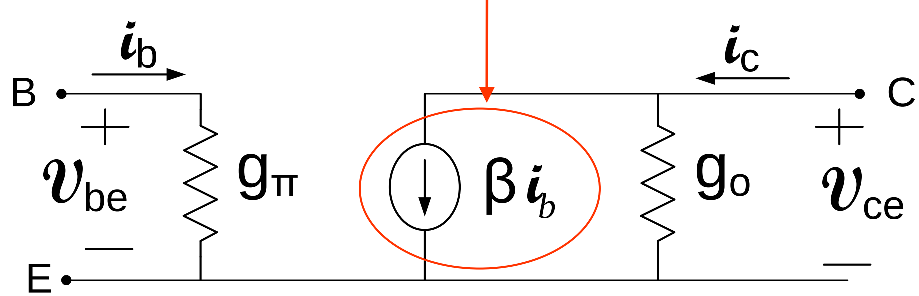
Review from last lecture

Small Signal BJT Model – alternate representation



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

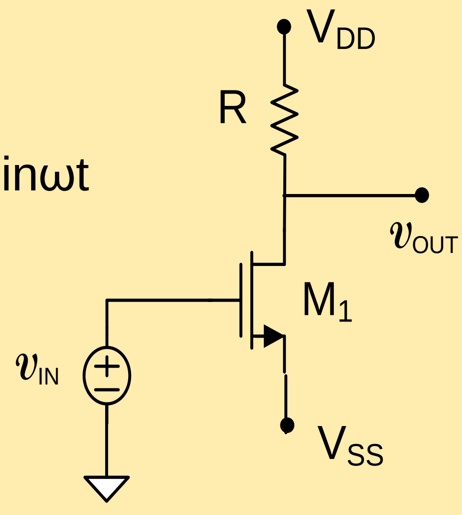
Alternate equivalent small signal model



$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Consider again: Review from last lecture

Small-signal analysis example



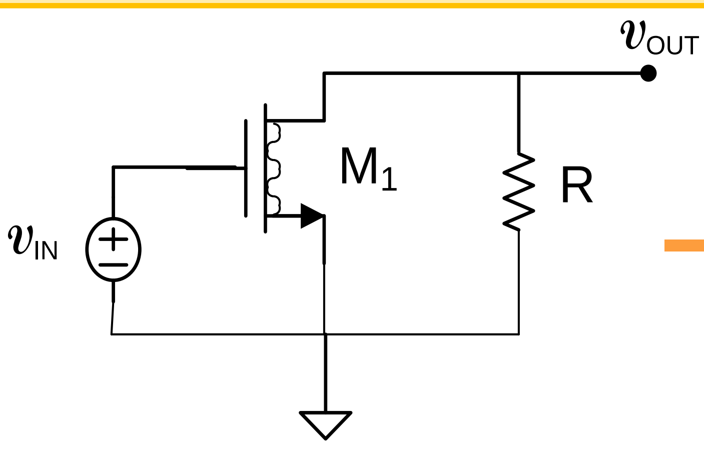
$v_{IN}=V_M\sin\omega t$

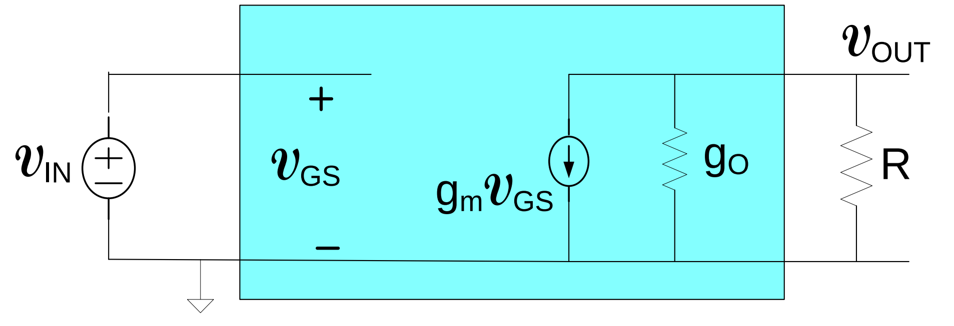
$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

Derived for $\lambda=0$ (equivalently $g_0=0$)

$$I_D=\mu C_{OX}\frac{W}{2L}(V_{GS}-V_T)^2$$

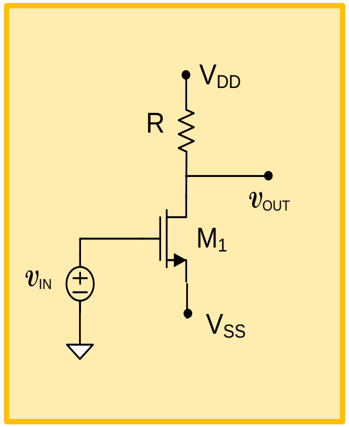
Recall the derivation was very tedious and time consuming!

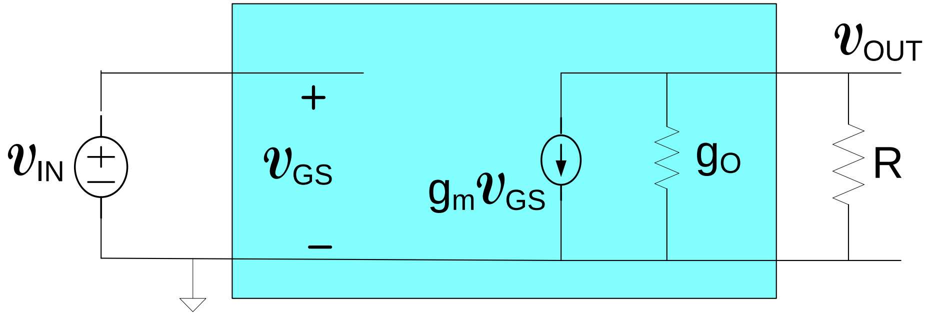




Consider again: Review from last lecture

Small-signal analysis example

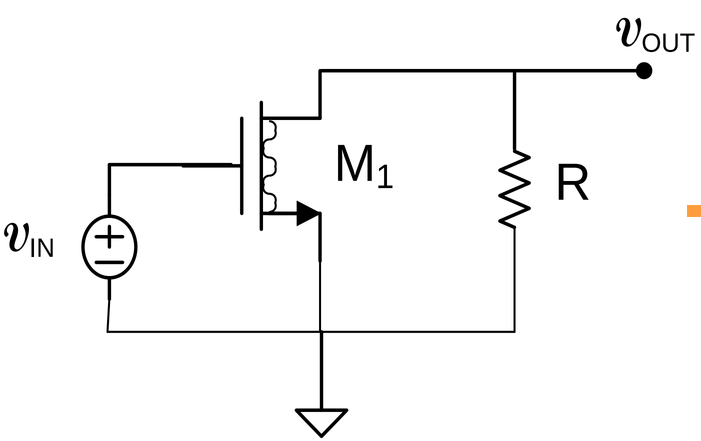




$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

This gain is expressed in terms of small-signal model parameters

For $\lambda=0$, $g_O = \lambda I_{DQ} = 0$



$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

but

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

$V_{GSQ} = -V_{SS}$

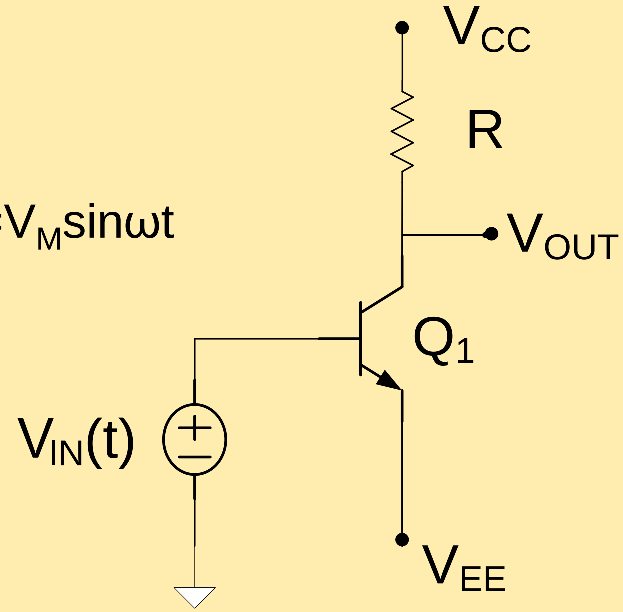
thus

$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

Consider again: Review from last lecture

Small signal analysis example

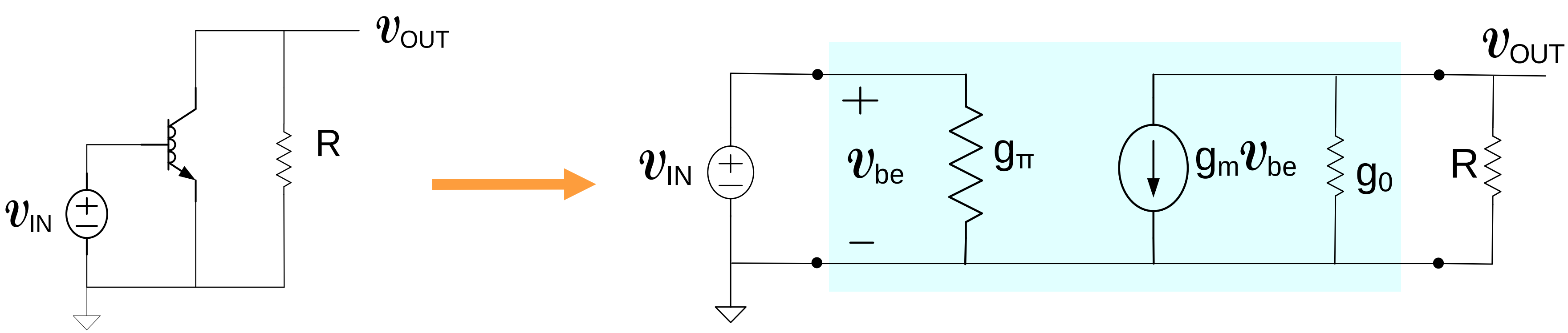
$v_{IN}=V_M \sin\omega t$



$$A_{VB} = -\frac{I_{CQ} R}{V_t}$$

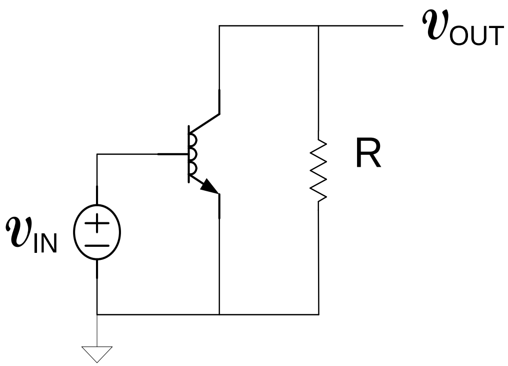
Derived for $V_{AF}=0$ (equivalently $g_o=0$)

Recall the derivation was very tedious and time consuming!



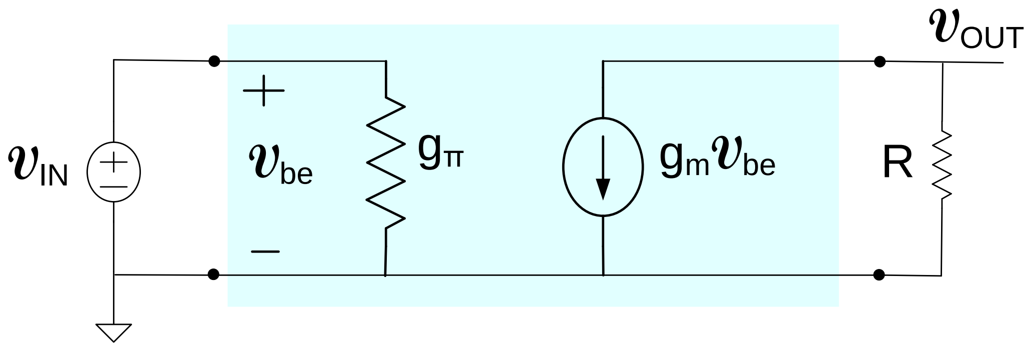
ss circuit

Review from last lecture



Neglect V_{AF} effects (i.e. $V_{AF}=\infty$) to be consistent with earlier analysis

$$g_o = \frac{I_{CQ}}{V_{AF}} \underset{V_{AF}=\infty}{=} 0$$



$$\left.\begin{aligned} v_{OUT} &= -g_m R v_{BE} \\ v_{IN} &= v_{BE} \end{aligned}\right\}$$

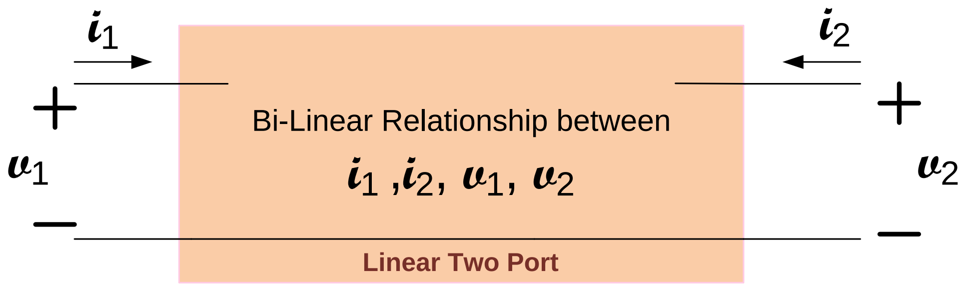
$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

$$g_m = \frac{I_{CQ}}{V_t}$$

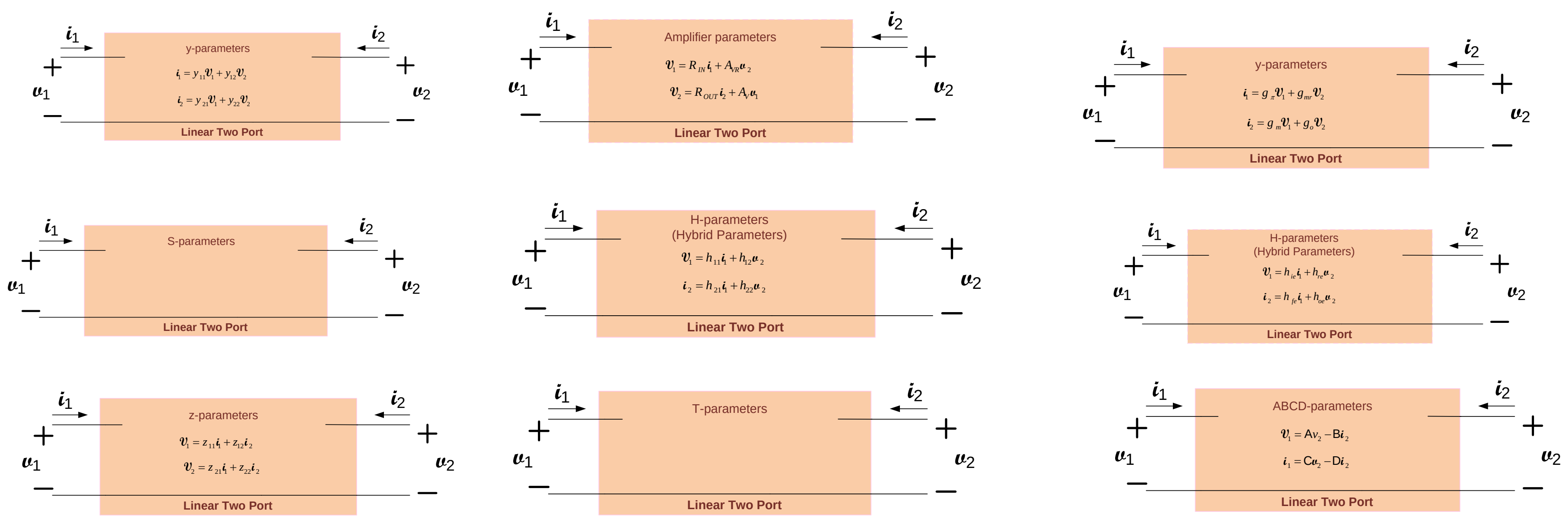
$$A_V = -\frac{I_{CQ} R}{V_t}$$

Note this is identical to what was obtained with the direct nonlinear analysis

Small-Signal Model Representations



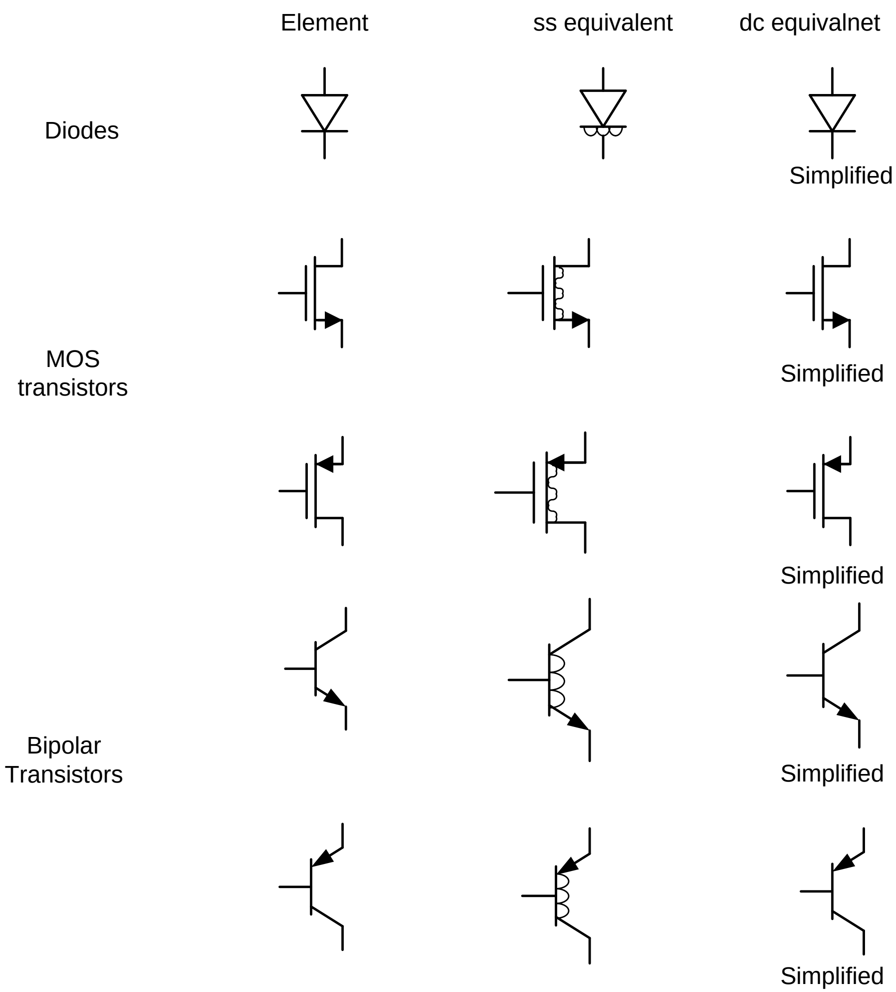
The good, the bad, and the **unnecessary** !!



- Equivalent circuits often given for each representation
- All provide identical characterization
- Easy to move from any one to another

Review from last lecture

Active Device Model Summary



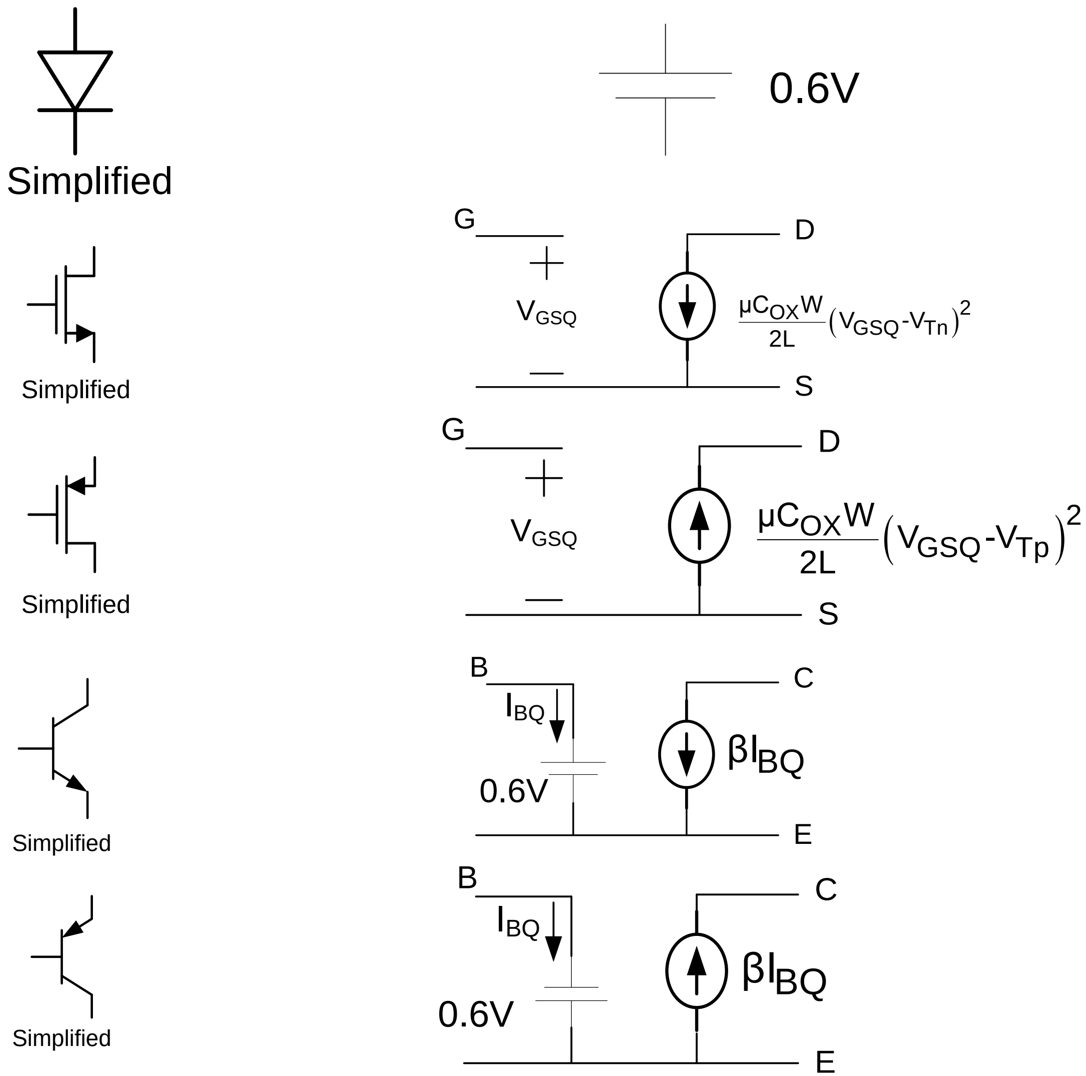
What are the simplified dc equivalent models?

Review from last lecture

Active Device Model Summary

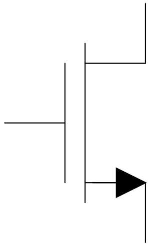
What are the simplified dc equivalent models?

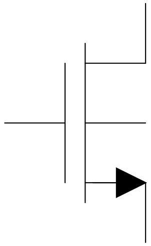
dc equivalent



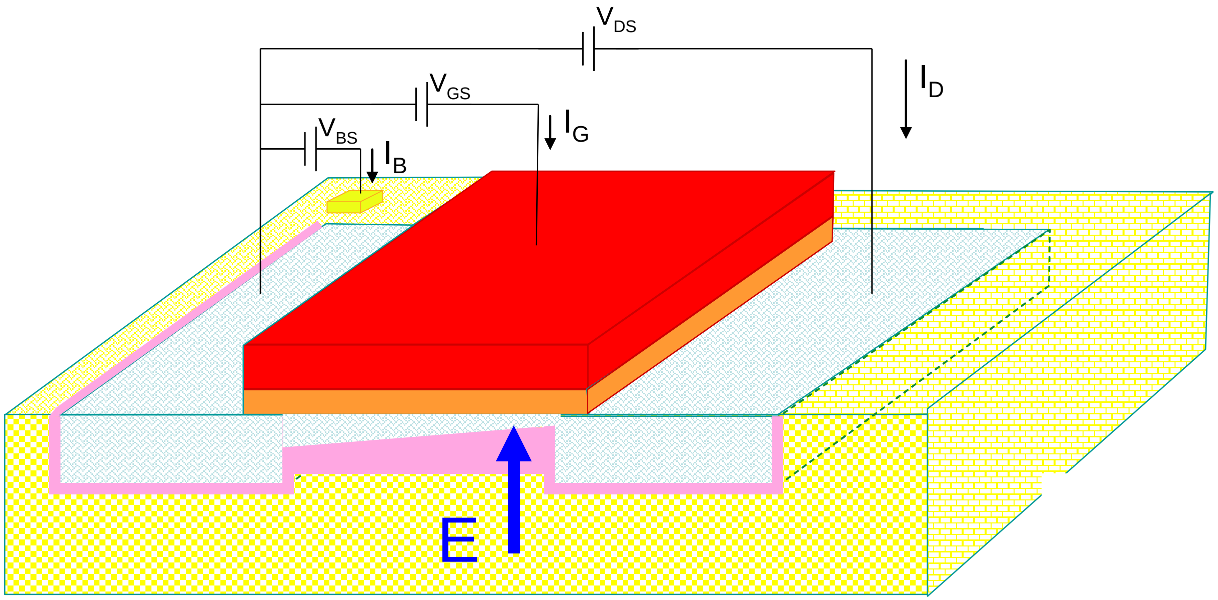
Small-Signal MOSFET Model Extension

Existing 3-terminal small-signal model does not depend upon the bulk voltage !





Recall that changing the bulk voltage changes the electric field in the channel region and thus the threshold voltage!

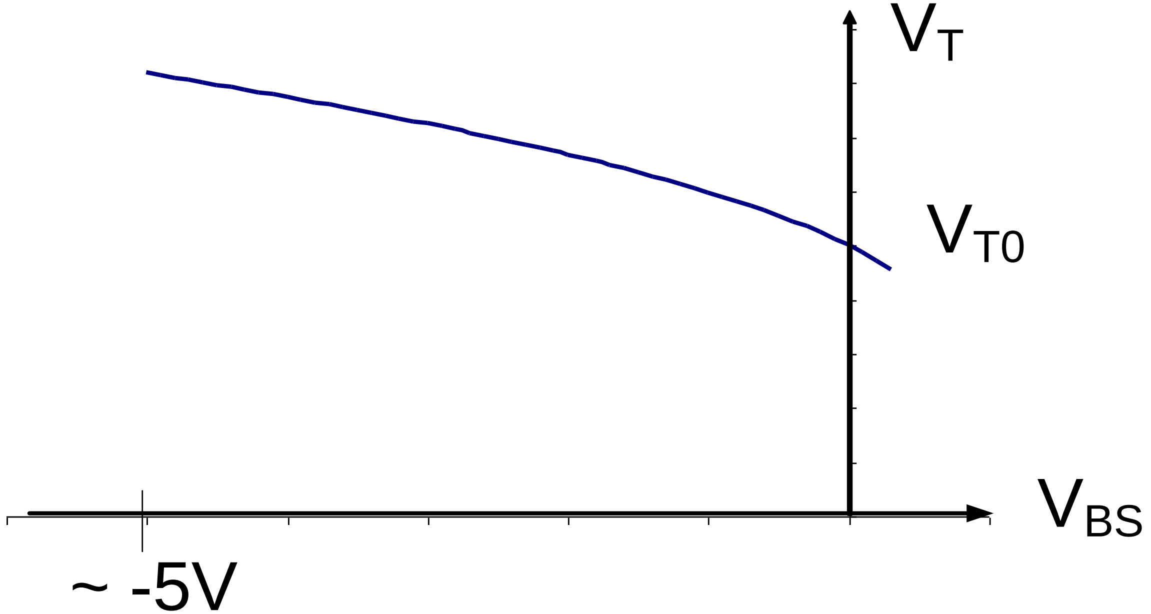


Recall:

Typical Effects of Bulk on Threshold Voltage for n-channel Device

$$V_T = V_{T0} + \gamma\left[\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right]$$

$$\gamma \cong 0.4\mathrm{V}^{-\frac{1}{2}} \qquad \phi \cong 0.6\mathrm{V}$$



Bulk-Diffusion Generally Reverse Biased (V_{BS}< 0 or at least less than 0.3V) for n-channel

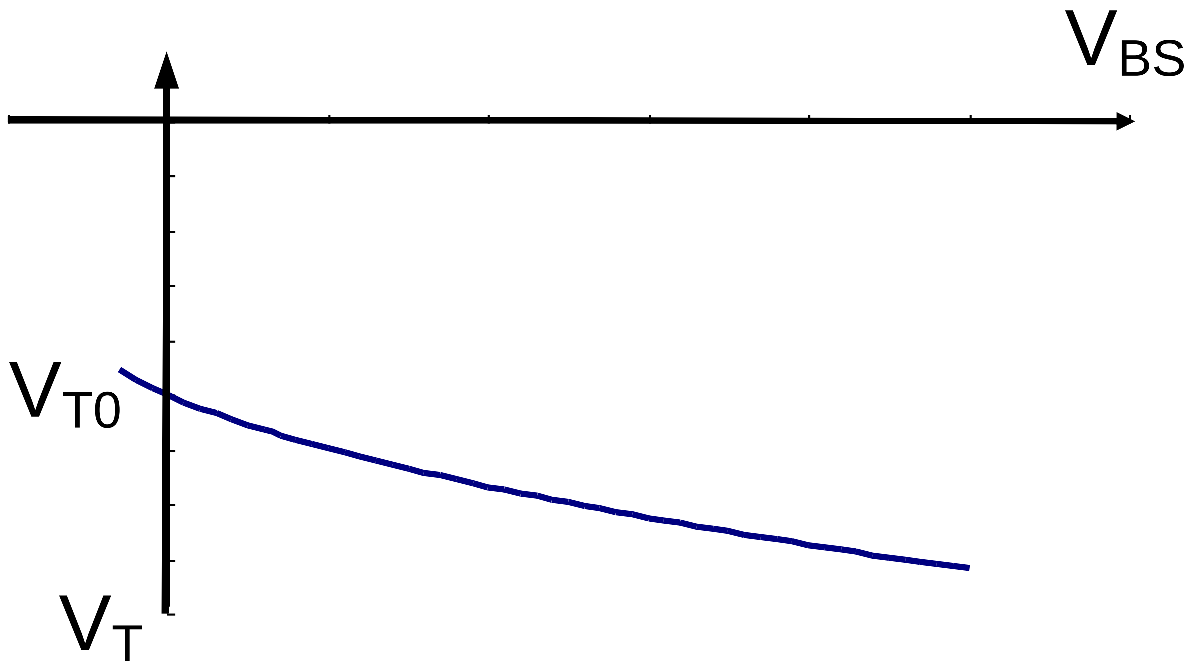
Shift in threshold voltage with bulk voltage can be substantial

Often V_{BS}=0

Recall: Typical Effects of Bulk on Threshold Voltage for p-channel Device

$$V_T = V_{T0} - \gamma\left[\sqrt{\phi + V_{BS}} - \sqrt{\phi}\right]$$

$\gamma \cong 0.4\text{V}^{-\frac{1}{2}}$ $\qquad \phi \cong 0.6\text{V}$

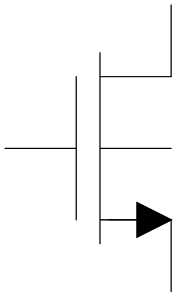


Bulk-Diffusion Generally Reverse Biased (V_{BS} > 0 or at least greater than -0.3V) for n-channel

Same functional form as for n-channel devices but V_{T0} is now negative and the magnitude of V_T still increases with the magnitude of the reverse bias

Recall:

4-terminal model extension



$$I_G = 0$$

$$I_B = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}, \varphi, \gamma, \lambda\}$

Design Parameters : {W,L} but only one degree of freedom W/L

biasing or quiescent point

Small-Signal 4-terminal Model Extension

$$I_G = 0$$

$$I_B = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$\mathbf{y_{11}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{12}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{13}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = g_m \qquad y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = g_o \qquad y_{23} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = g_{mb}$$

$$\mathbf{y_{31}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{32}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{33}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

Small-Signal 4-terminal Model Extension

$$I_D = \mu C_{OX}\frac{W}{2L}(V_{GS}-V_T)^2\bullet(1+\lambda V_{DS})$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Definition:

$$V_{EB} = V_{GS} - V_T$$

$$V_{EBQ} = V_{GSQ} - V_{TQ}$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = \left.\mu C_{OX}\frac{W}{\not{2}L}\not{2}(V_{GS}-V_T)^1\bullet(1+\lambda V_{DS})\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX}\frac{W}{L}V_{EBQ}$$

Same as 3-term

$$g_o = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = \left.\mu C_{OX}\frac{W}{2L}2(V_{GS}-V_T)^2\bullet\lambda\right|_{\vec{V}=\vec{V}_Q} \cong \lambda I_{DQ}$$

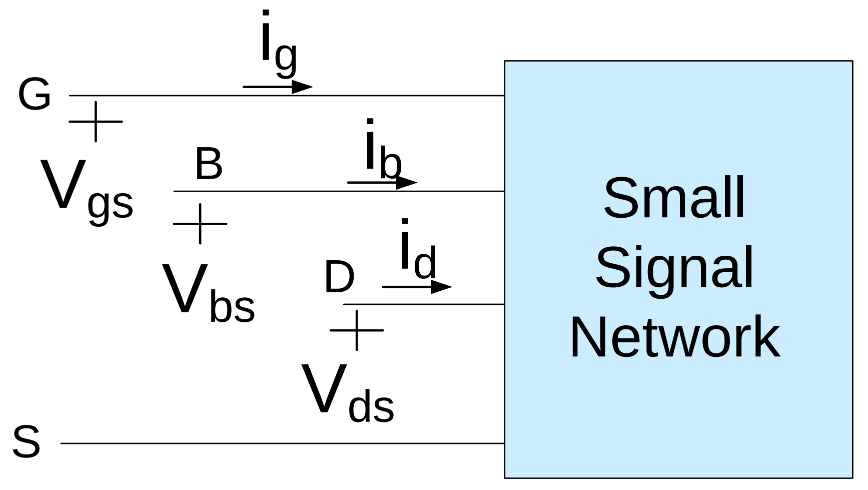
Same as 3-term

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \left.\mu C_{OX}\frac{W}{\not{2}L}\not{2}(V_{GS}-V_T)^1\bullet\left(-\frac{\partial V_T}{\partial V_{BS}}\right)\bullet(1+\lambda V_{DS})\right|_{\vec{V}=\vec{V}_Q}$$

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX}\frac{W}{L}V_{EBQ}\bullet\left.\frac{\partial V_T}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \left(\mu C_{OX}\frac{W}{L}V_{EBQ}\right)(-\not{1})\gamma\frac{1}{2}\left.(\phi - V_{BS})^{-\frac{1}{2}}\right|_{\vec{V}=\vec{V}_Q}(-\not{1})$$

$$g_{mb} \cong g_m\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}$$

Small Signal 4-terminal MOSFET Model Summary



$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ}$$

$$i_g = 0$$

$$g_o = \lambda I_{DQ}$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

Relative Magnitude of Small Signal MOS Parameters

Consider:

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

3 alternate equivalent expressions for g_m

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \qquad g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}} \qquad g_m = \frac{2I_{DQ}}{V_{EBQ}}$$

Consider, as an example:

$\mu C_{OX}=100\mu A/V^2$, $\lambda=.01V^{-1}$, $\gamma = 0.4V^{0.5}$, $V_{EBQ}=1V$, $W/L=1$, $V_{BSQ}=0V$

$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L} V_{EBQ}^2 = \frac{10^{-4} \not{W}}{2\not{L}} (1V)^2 = 5E\text{-}5$$

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} = 1E\text{-}4$$

$$g_o = \lambda I_{DQ} = 5E\text{-}7$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) = .26 g_m$$

In this example

$$g_0 << g_m, g_{mb}$$

$$g_{mb} < g_m$$

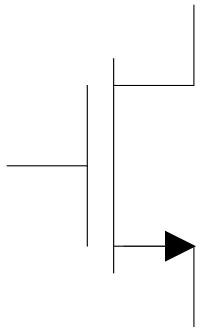
This relationship is common

In many circuits, $v_{BS}=0$ as well

- **Often the g_o term can be neglected in the small signal model because it is so small**
- **Be careful about neglecting g_o prior to obtaining a final expression**

Large and Small Signal 4-Terminal MOSFET Model Summary

Large Signal Model

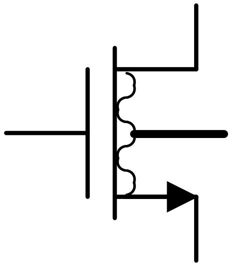


$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

saturation

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Small Signal Model



saturation

$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

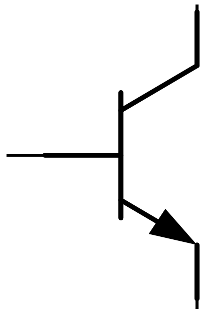
$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ}$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

$$g_o = \lambda I_{DQ}$$

Large and Small Signal BJT Model Summary

Large Signal Model



$$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$V_{BE}>0.4V$

$V_{BC}<0$

Forward Active

$V_{BE}=0.7V$
$V_{CE}=0.2V$

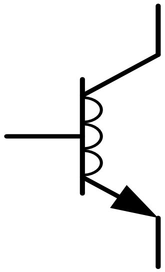
$I_C<\beta I_B$

$I_C=I_B=0$

$V_{BE}<0$

$V_{BC}<0$

Small Signal Model



Forward Active

$$i_b = g_\pi v_{be}$$

$$i_c = g_m v_{be} + g_0 v_{ce}$$

where

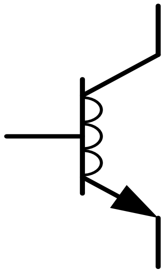
$$g_m = \frac{I_{CQ}}{V_t}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Relative Magnitude of Small Signal BJT Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$



$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]}$$

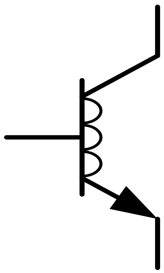
$$\frac{g_\pi}{g_o} = \frac{\left[\dfrac{I_Q}{\beta V_t}\right]}{\left[\dfrac{I_Q}{V_{AF}}\right]}$$

$$g_m >> g_\pi >> g_o$$

Often the g_o term can be neglected in the small signal model because it is so small

Relative Magnitude of Small Signal Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$



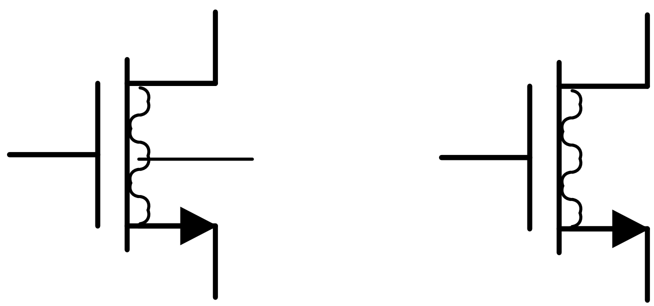
$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]} = \beta$$

$$\frac{g_\pi}{g_o} = \frac{\left[\dfrac{I_Q}{\beta V_t}\right]}{\left[\dfrac{I_Q}{V_{AF}}\right]} = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \bullet 26mV} = 77$$

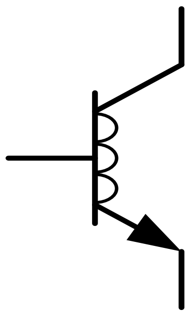
$$g_m >> g_\pi >> g_o$$

- **Often the g_o term can be neglected in the small signal model because it is so small**
- **Be careful about neglecting g_o prior to obtaining a final expression**

Small Signal Model Simplifications for the MOSFET and BJT



MOSFET

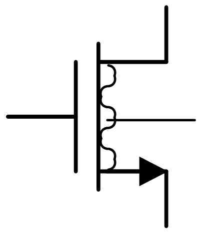


BJT

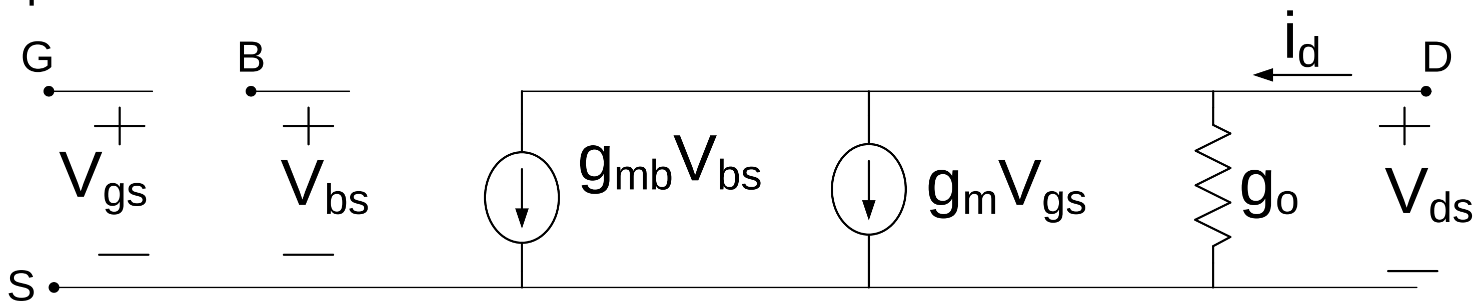
Often simplifications of the small signal model are adequate for a given application

These simplifications will be discussed next

Small Signal MOSFET Model Summary



An equivalent Circuit:



$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o = \lambda I_{DQ}$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

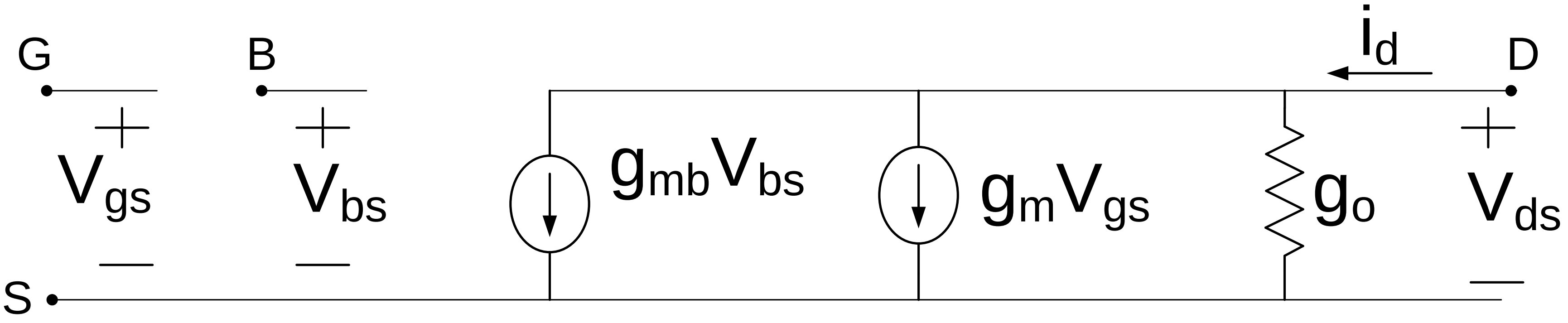
Alternate equivalent representations for g_m from $I_D \cong \mu C_{OX}\frac{W}{2L}\left(V_{GS} - V_T\right)^2$

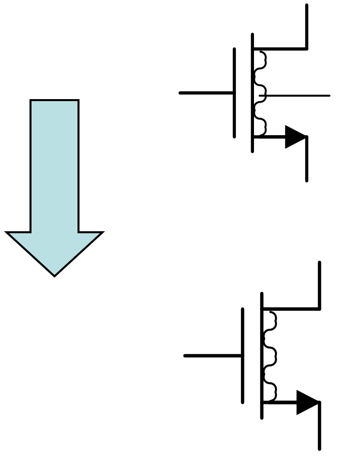
$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}} \qquad g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

$$g_{mb} < g_m \qquad g_0 << g_m, g_{mb}$$

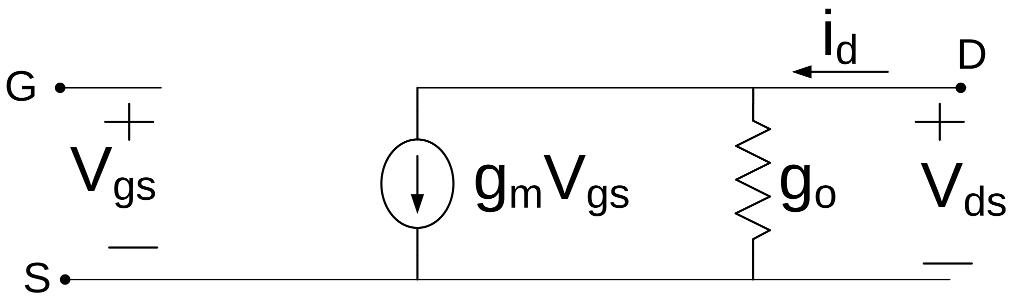
This contains absolutely no more information than the set of small-signal model equations

Small Signal Model Simplifications

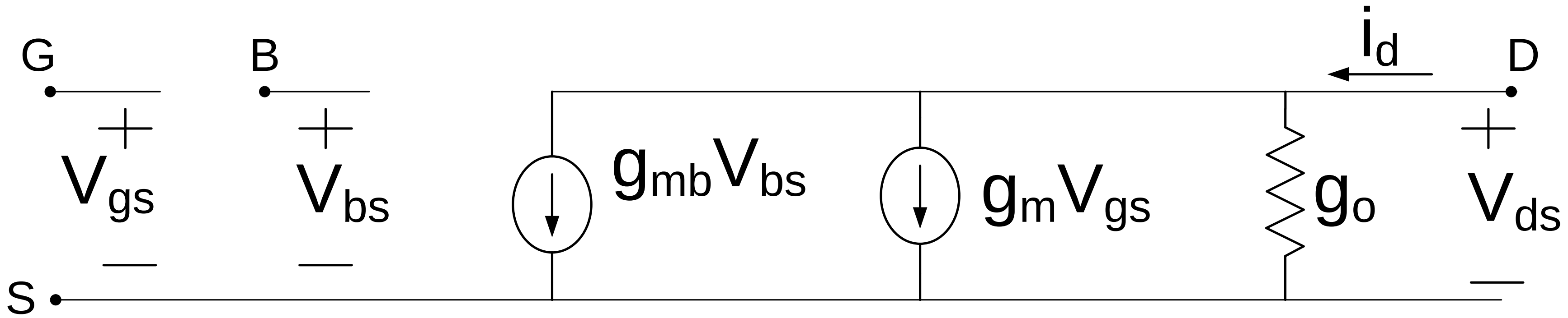


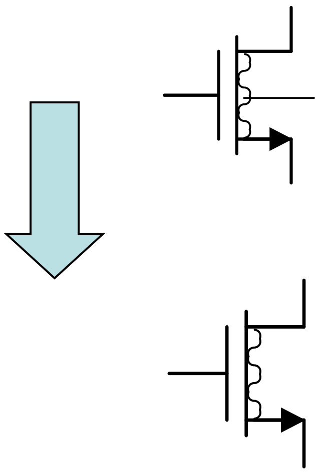


Simplification that is often adequate

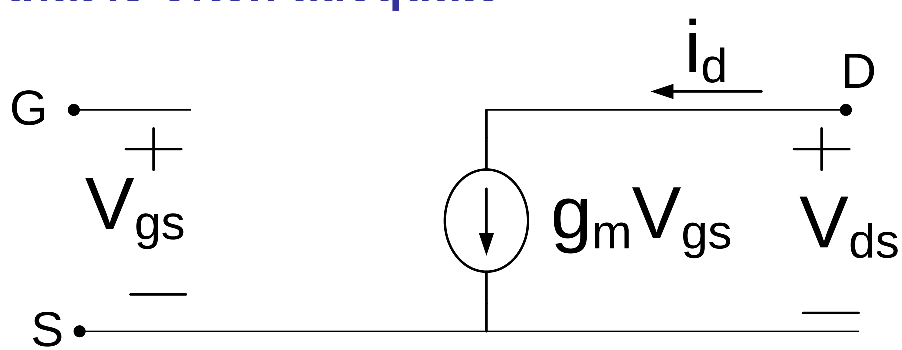


Small Signal Model Simplifications



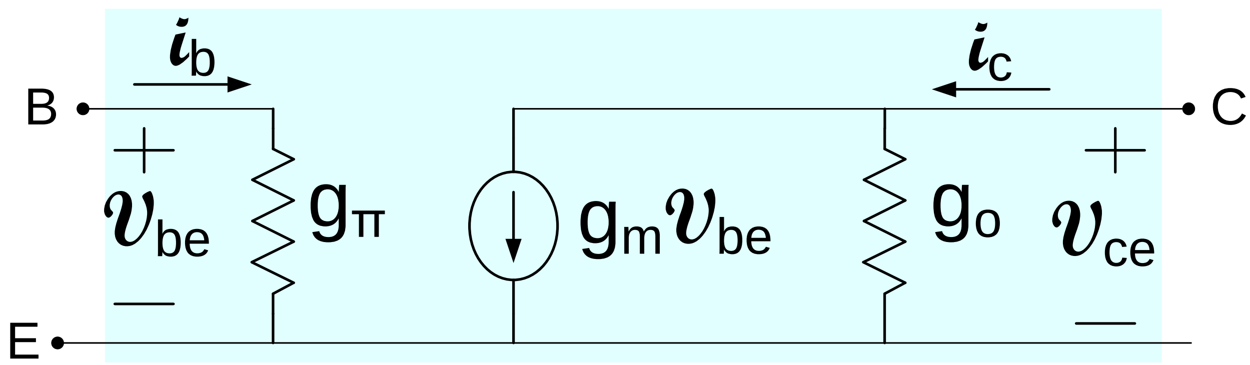


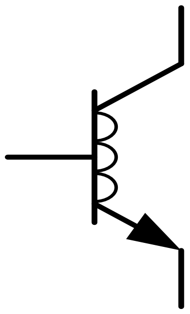
Even further simplification that is often adequate



Small Signal BJT Model Summary

An equivalent circuit



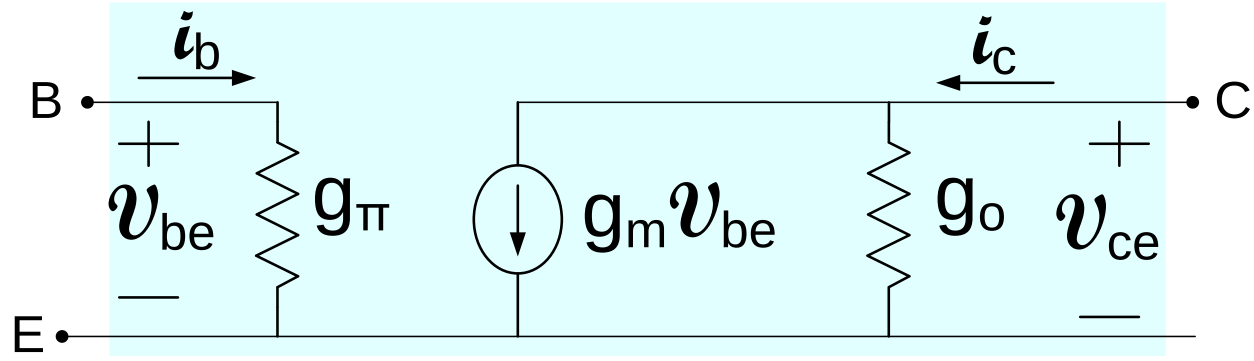


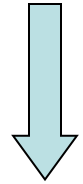
$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

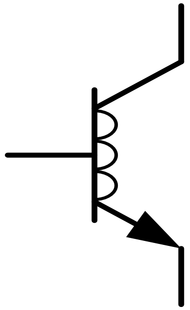
$$g_m >> g_\pi >> g_o$$

This contains absolutely no more information than the set of small-signal model equations

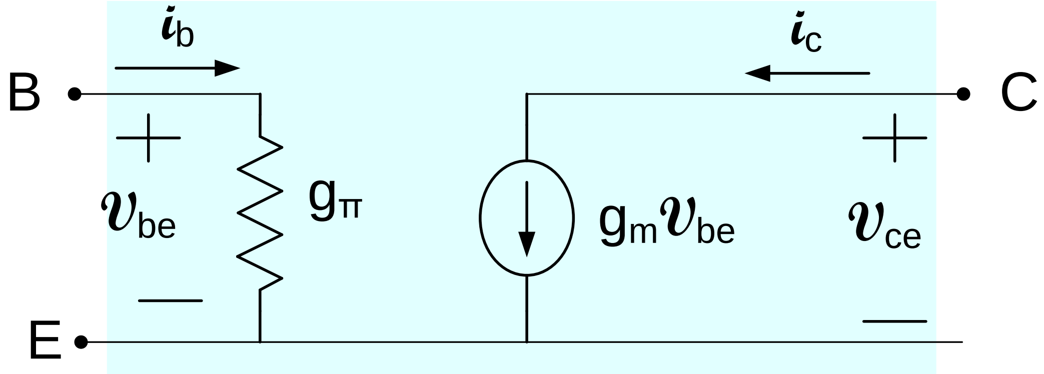
Small Signal BJT Model Simplifications





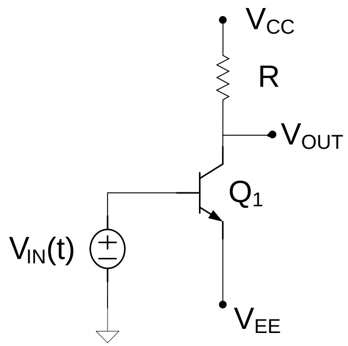


Simplification that is often adequate

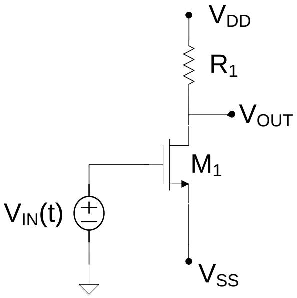


Gains for MOSFET and BJT Circuits

BJT



MOSFET

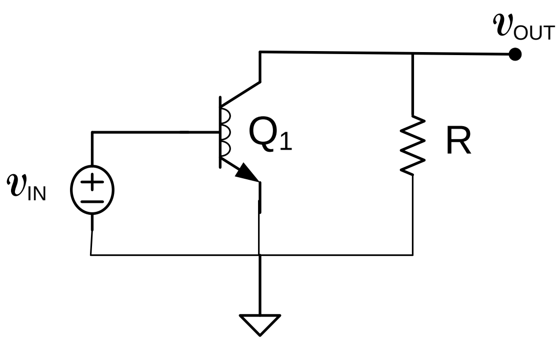


$$A_{VB} = -\frac{I_{CQ} R_1}{V_t}$$

Large Signal Parameter Domain

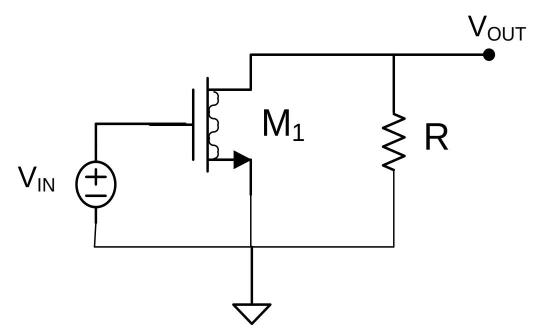
$$A_{VM} = \frac{2 I_{DQ} R}{[V_{SS} + V_T]}$$

For both circuits



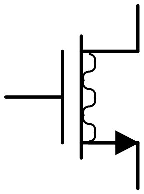
Small Signal Parameter Domain

$$A_V = -g_m R$$



- Gains are identical in small-signal parameter domain !
- Gains vary linearly with small signal parameter g_m
- Power is often a key resource in the design of an integrated circuit
- In both circuits, power is proportional to I_{CQ} , I_{DQ}

How does g_m vary with I_{DQ}?



$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

Varies with the square root of I_{DQ}

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with I_{DQ}

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

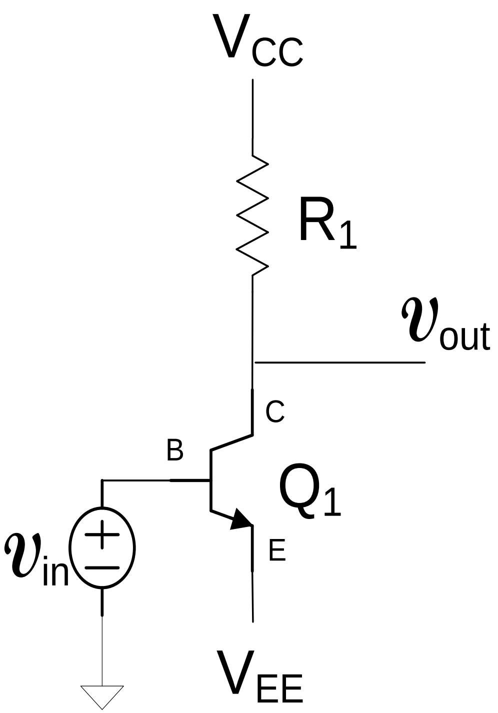
Doesn't vary with I_{DQ}

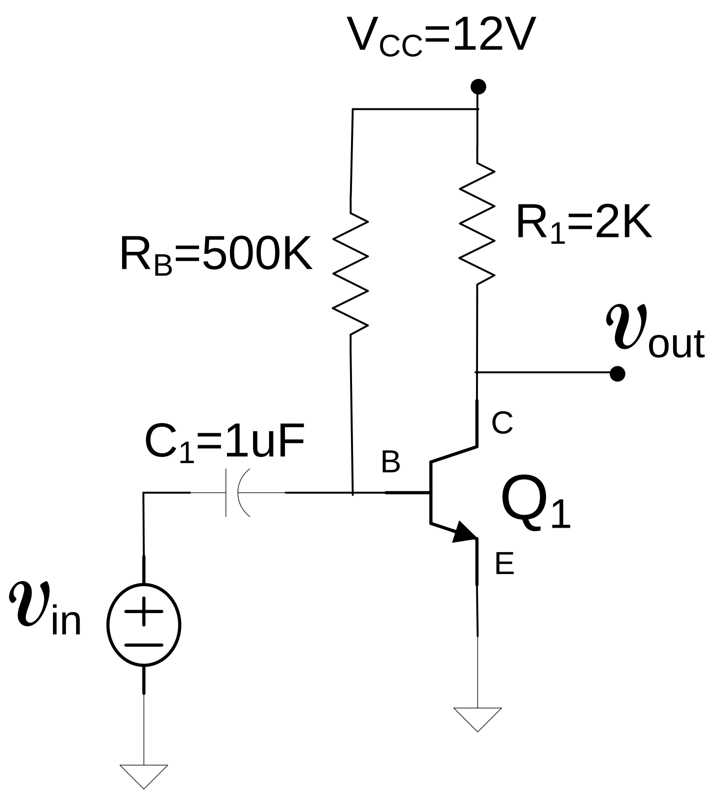
How does g_m vary with I_{DQ}?

All of the above are true – but with qualification

g_m is a function of more than one variable (I_{DQ}) and how it varies depends upon how the remaining variables are constrained

Amplifier Biasing (precursor)





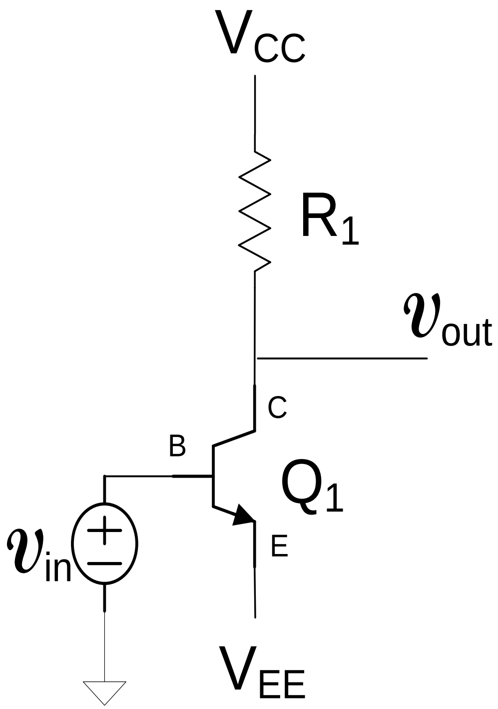
Not convenient to have multiple dc power supplies
V_{OUTQ} very sensitive to V_{EE}

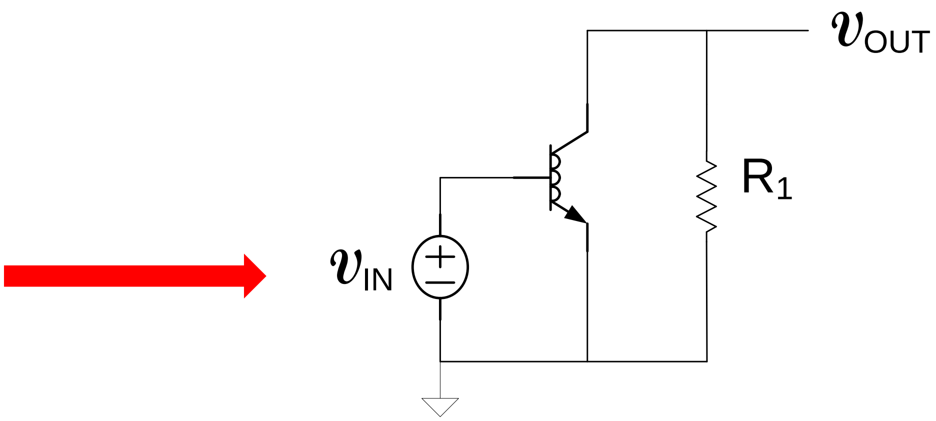
Single power supply
Additional resistor and capacitor

Compare the small-signal equivalent circuits of these two structures

Compare the small-signal voltage gain of these two structures

Amplifier Biasing (precursor)





V_{CC} and V_{EE} have disappeared !

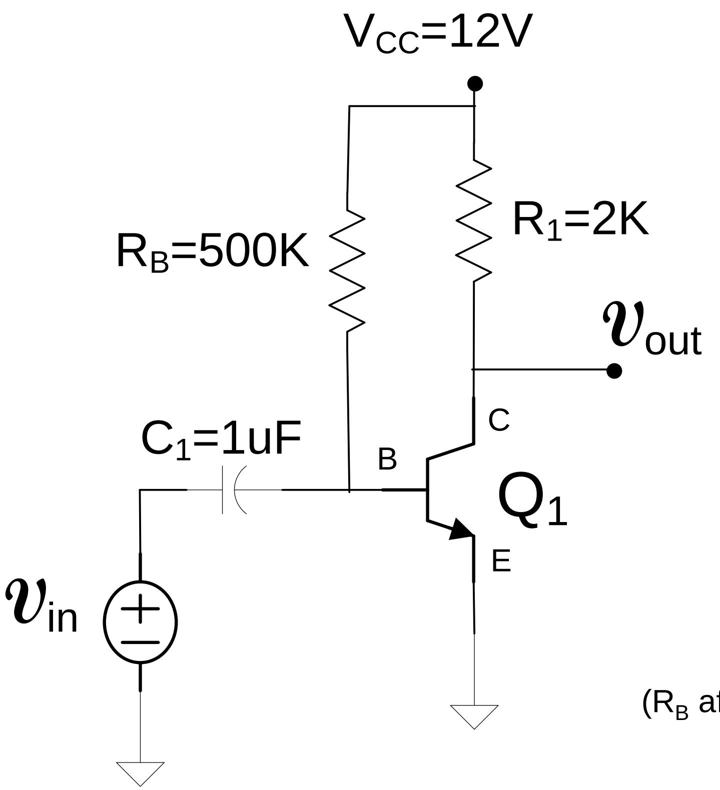
$$A_V \simeq -g_m R_1$$

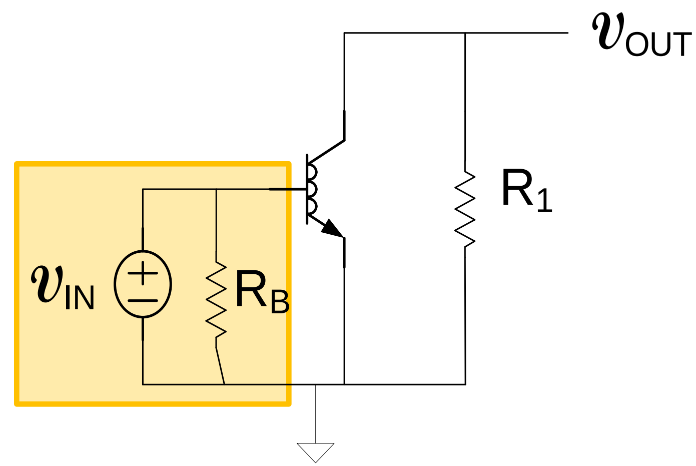
- Voltage sources V_{EE} and V_{CC} used for biasing
- Not convenient to have multiple dc power supplies
- V_{OUTQ} very sensitive to V_{EE}

➢ Biasing is used to obtain the desired operating point of a circuit

➢ Ideally the biasing circuit should not distract significantly from the basic operation of the circuit

Amplifier Biasing (precursor)





$$A_V \simeq -g_m R_1$$

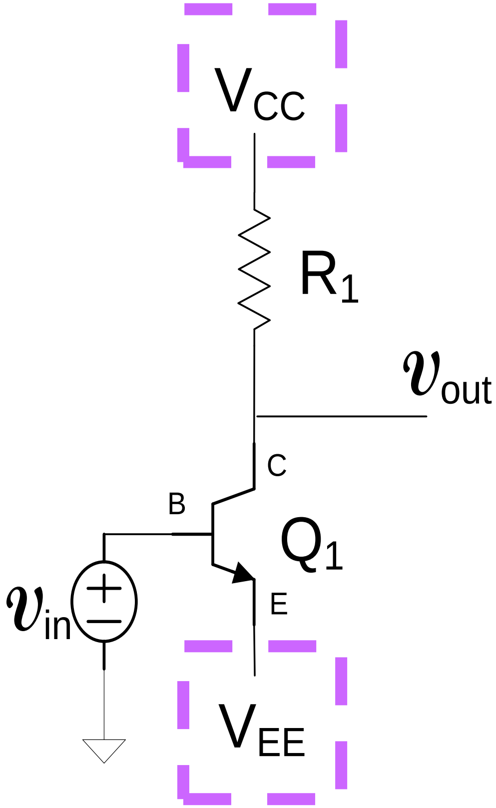
(R_B affects input impedance but not voltage gain)

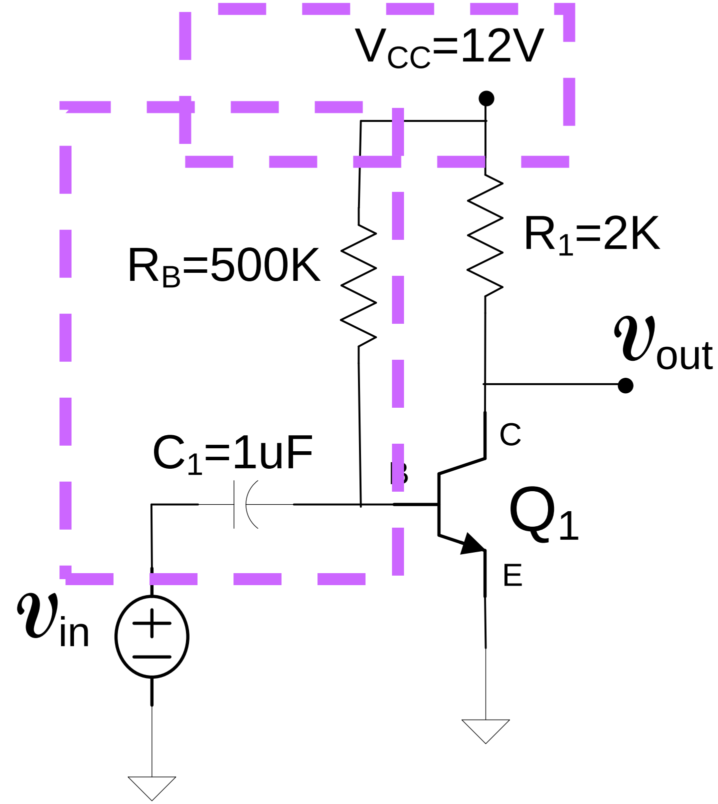
Single power supply
Additional resistor and capacitor
Thevenin Equivalent of v_{IN} & R_B is v_{IN}

- Biasing is used to obtain the desired operating point of a circuit
- Ideally the biasing circuit should not distract significantly from the basic operation of the circuit

Amplifier Biasing (precursor)

Biasing Circuits shown in purple

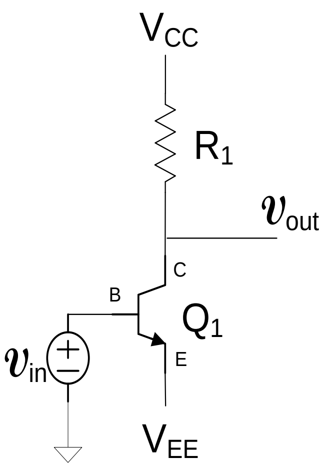


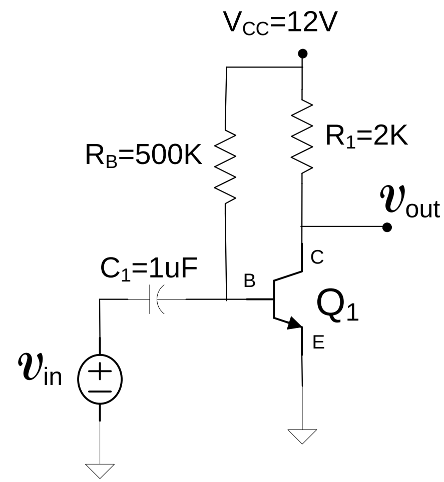


Not convenient to have multiple dc power supplies
V_{OUTQ} very sensitive to V_{EE}

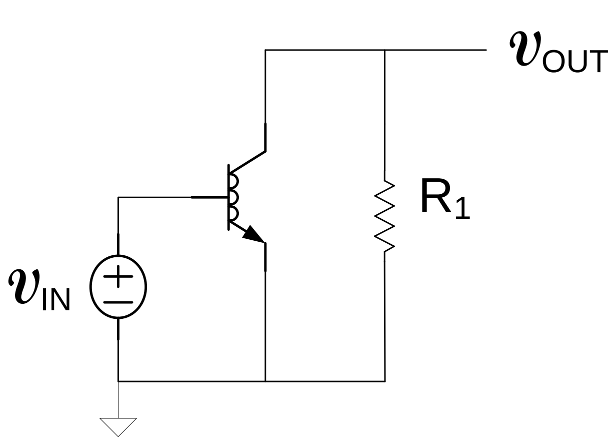
Single power supply
Additional resistor and capacitor

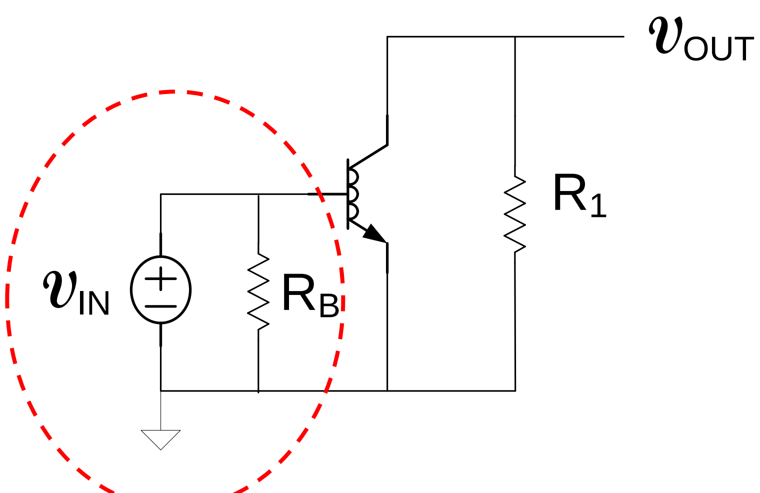
Amplifier Biasing (precursor)





Compare the small-signal equivalent circuits of these two structures





Since Thevenin equivalent circuit in red circle is V_{IN}, both circuits have same voltage gain

But the load placed on V_{IN} is different

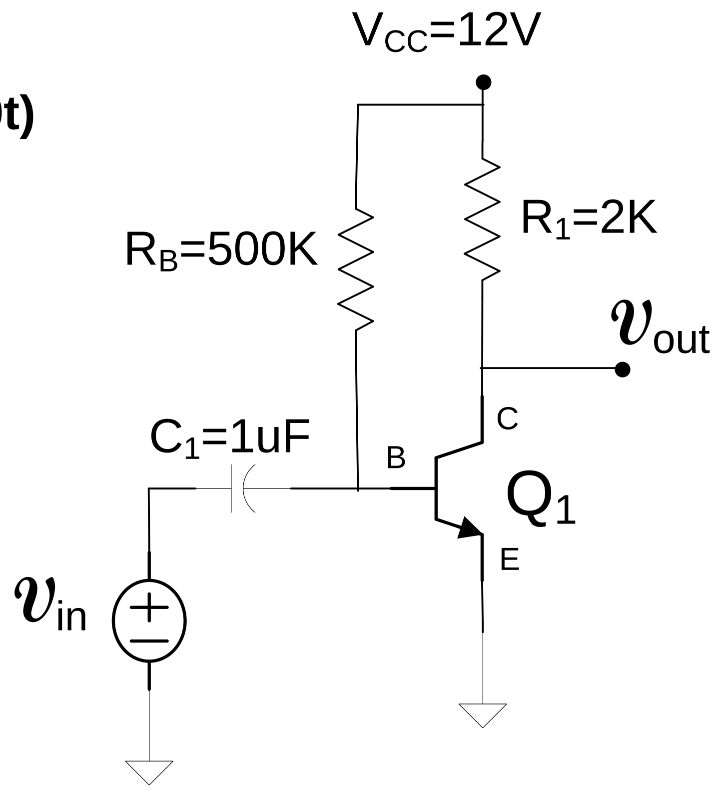
Method of characterizing the amplifiers is needed to assess impact of difference

Amplifier Characterization (an example)

This example serves as a precursor to amplifier characterization

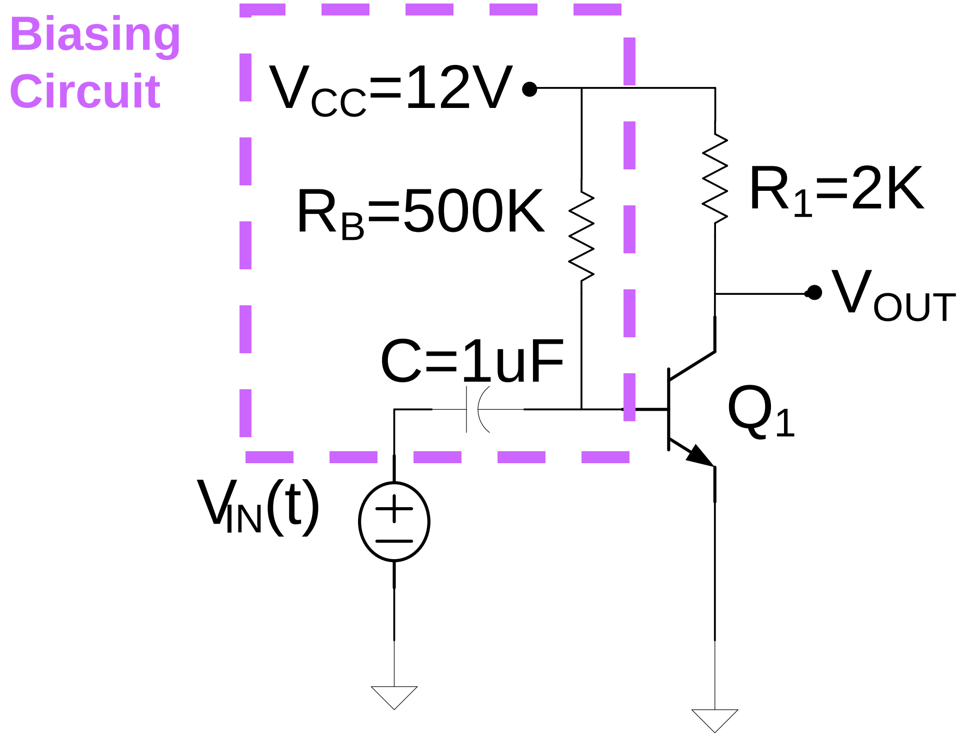
Determine V_{OUTQ}, A_V, R_{IN} Assume β=100

Determine v_{OUT} and $V_{OUT}(t)$ if v_{IN}=.002sin(400t)



In the following slides we will analyze this circuit

Amplifier Characterization (an example)

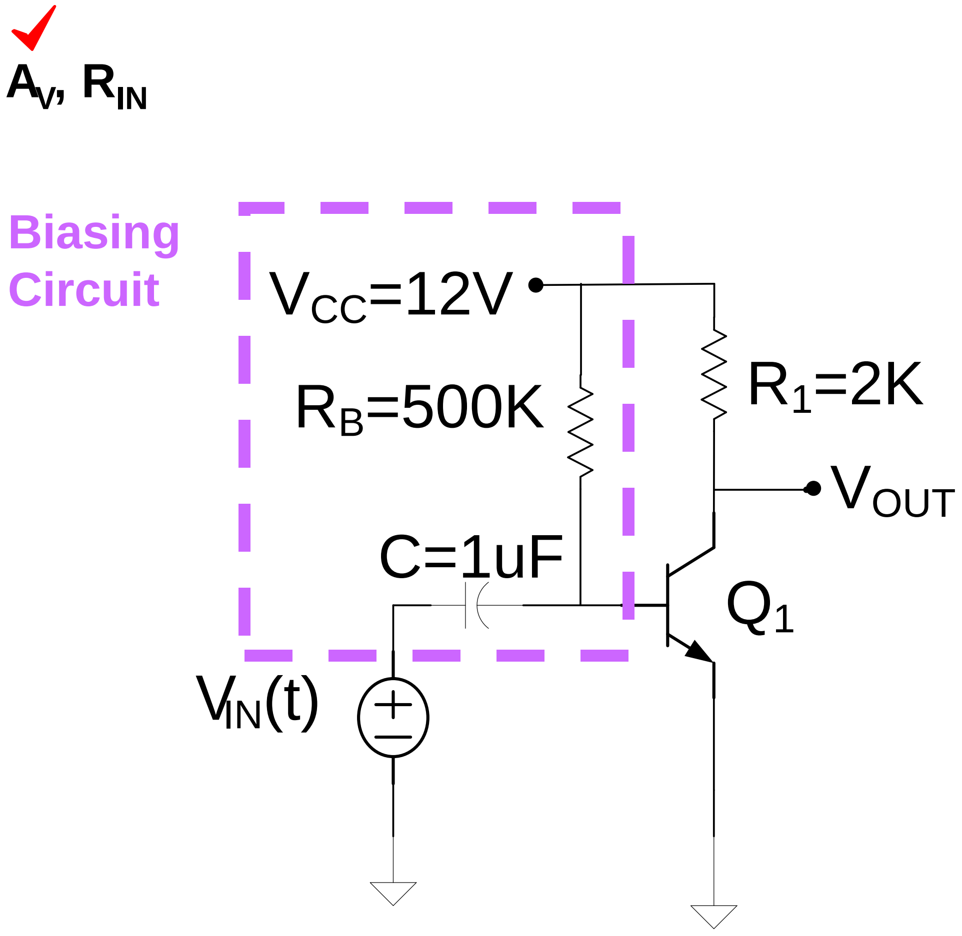


(biasing components: C, R_B, V_{CC} in this case, all disappear in small-signal gain circuit)

Several different biasing circuits can be used

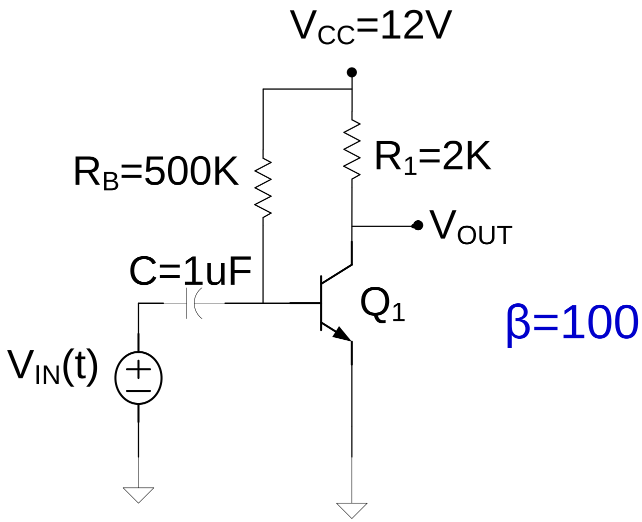
Amplifier Characterization (an example)

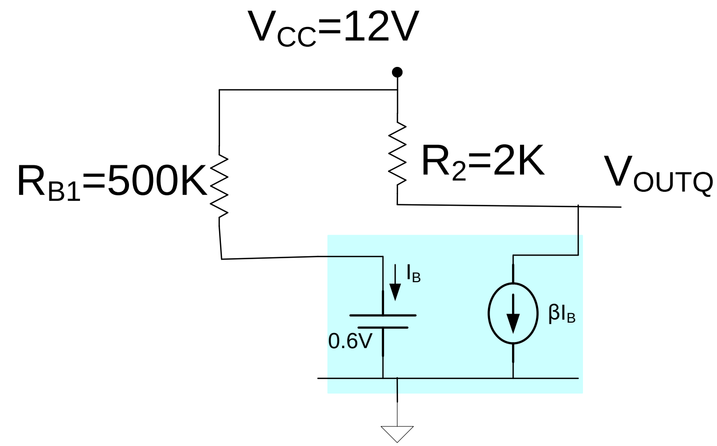
Determine V_{OUTQ}, A_V, R_{IN}



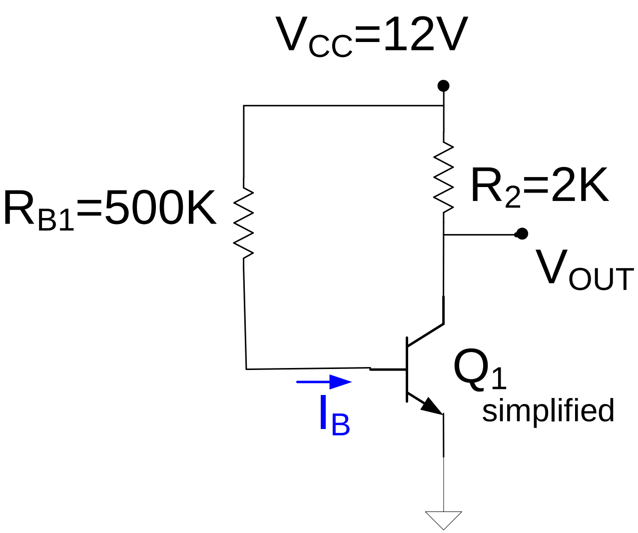
Amplifier Characterization (an example)

Determine V_{OUTQ}





dc equivalent circuit



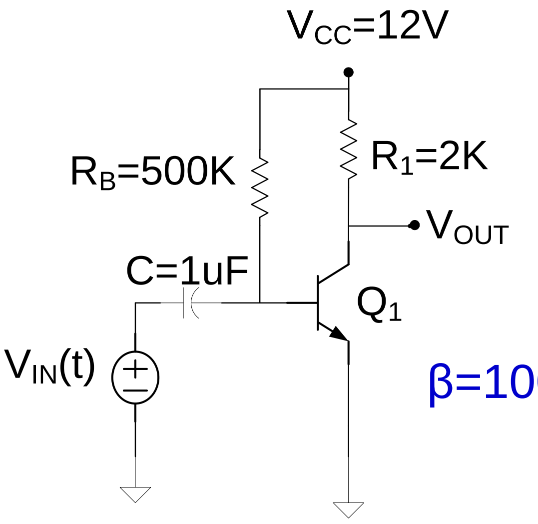
dc equivalent circuit

$$I_{CQ} = \beta I_{BQ} = 100\left(\frac{12V-0.6V}{500K}\right) = 2.3mA$$

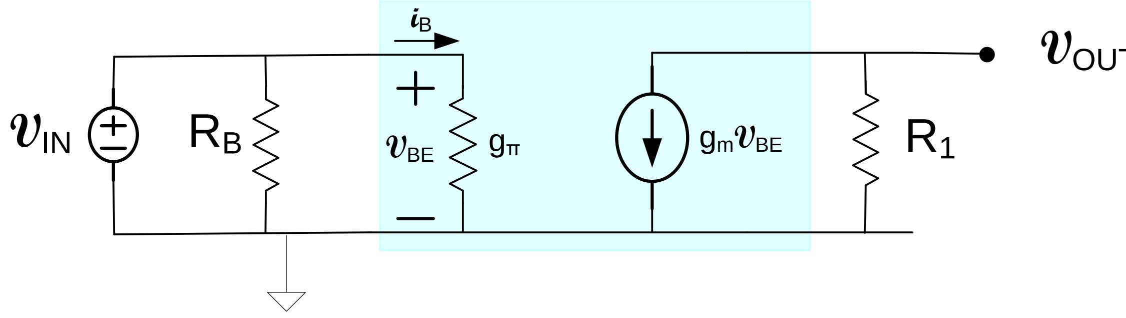
$$V_{OUTQ} = 12V - I_{CQ}R_1 = 12V - 2.3mA \bullet 2K = 7.4V$$

Amplifier Characterization (an example)

Determine the SS voltage gain (A_V)



β=100



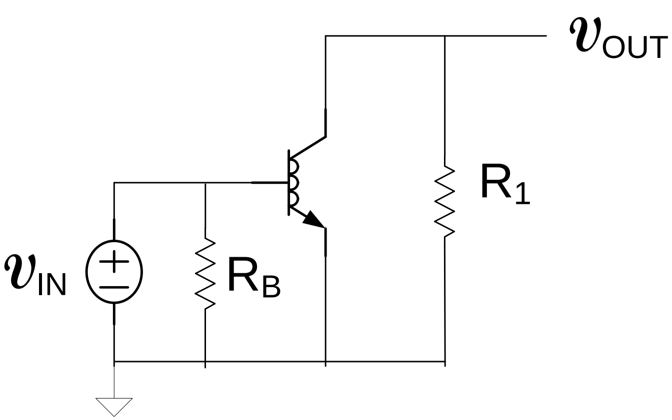
ss equivalent circuit

$$\left.\begin{aligned} v_{OUT} &= -g_m v_{BE} R_1 \\ v_{IN} &= v_{BE} \end{aligned}\right\}$$

$$A_V = -R_1 g_m$$

$$A_V \cong -\frac{I_{CQ} R_1}{V_t}$$

$$A_V \cong -\frac{2.3\text{mA} \bullet 2\text{K}}{26\text{mV}} \cong -177$$



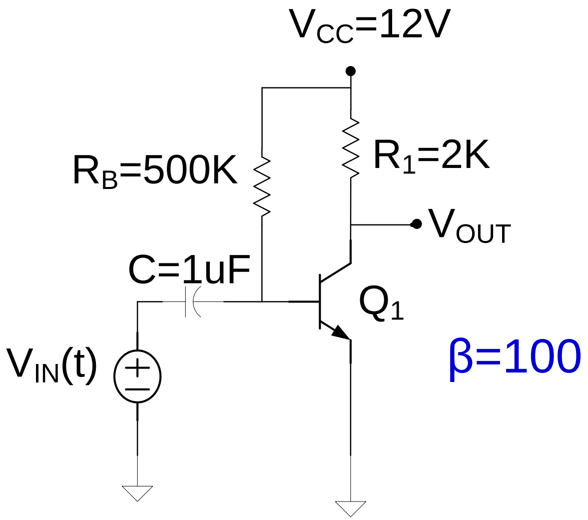
ss equivalent circuit

This basic amplifier structure is widely used and repeated analysis serves no useful purpose

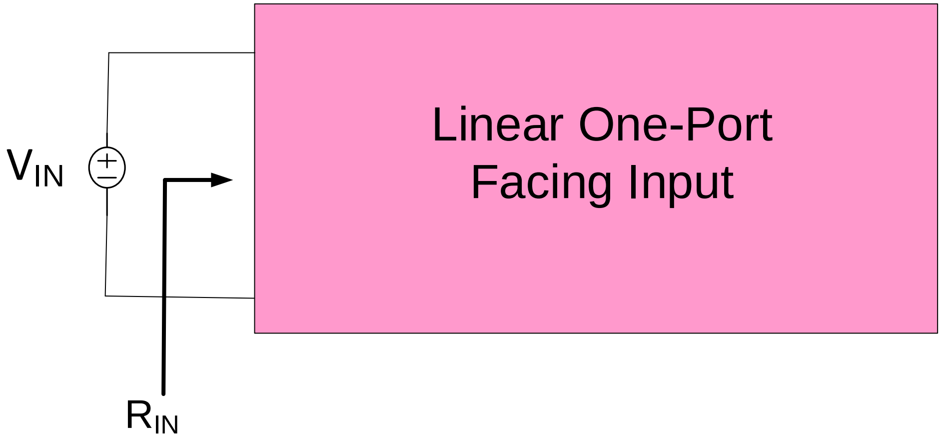
Have seen this circuit before but will repeat for review purposes

Amplifier Characterization (an example)

Determine V_{OUTQ}, A_V, R_{IN} ✓

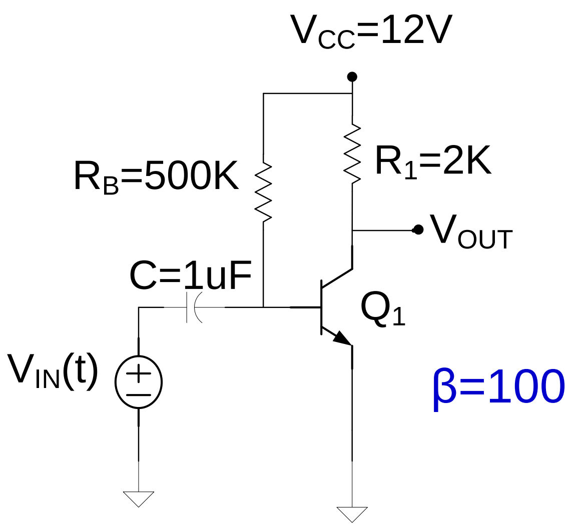


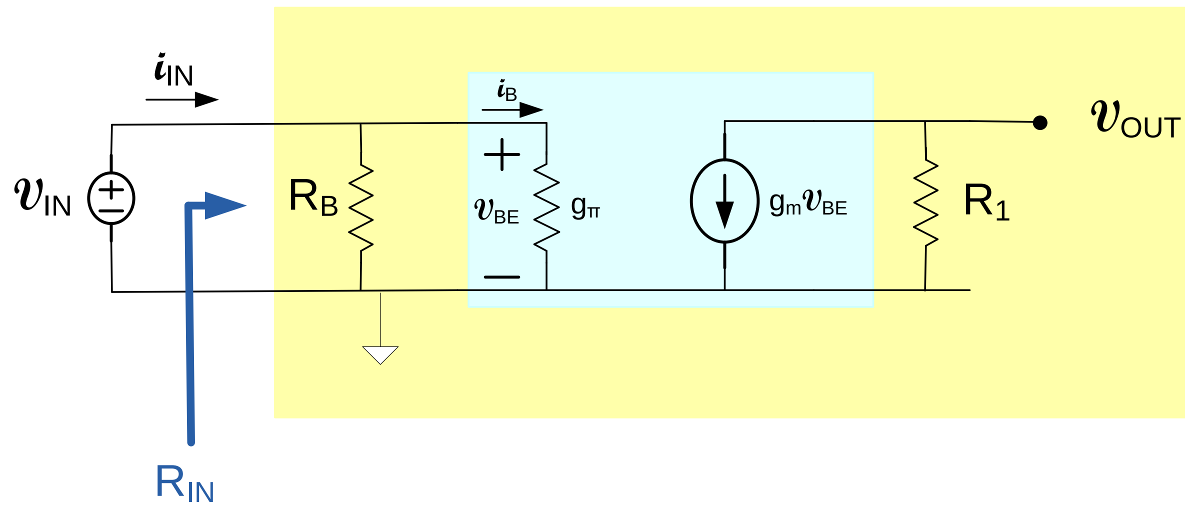
- Here R_{IN} is defined to be the impedance facing V_{IN}
- Here any load is assumed to be absorbed into the one-port
- Later will consider how load is connected in defining R_{IN}

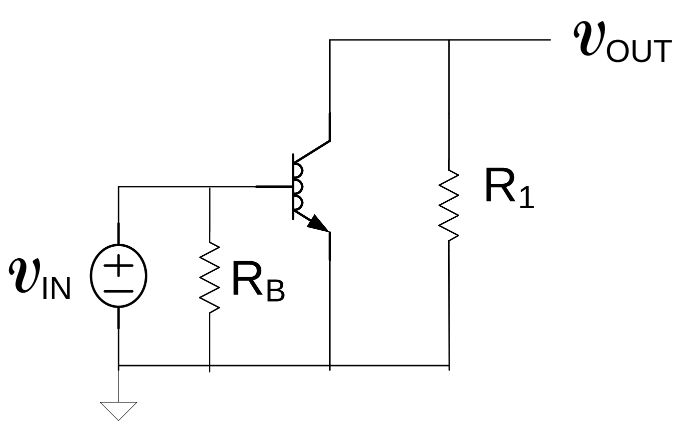


Amplifier Characterization (an example)

Determine R_{IN}







ss equivalent circuit

$$R_{in} = \frac{v_{IN}}{i_{IN}}$$

$$R_{in} = R_B // r_\pi$$

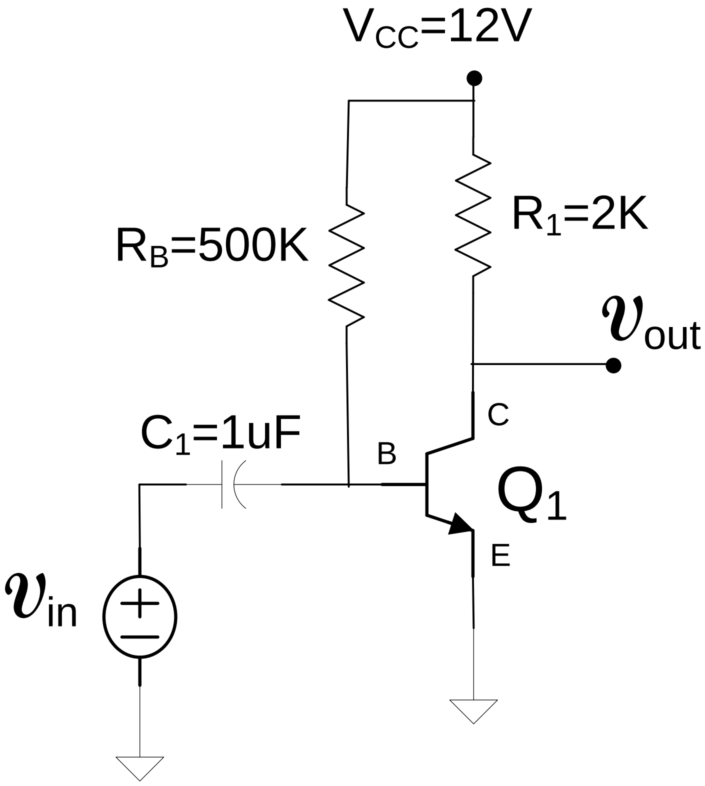
Usually $R_B >> r_\pi$

$$R_{in} = R_B // r_\pi \cong r_\pi$$

$$R_{in} \cong r_\pi = \frac{I_{CQ}}{\beta V_t}$$

Amplifier Characterization (an example)

Determine v_{OUT} and $V_{OUT}(t)$ if v_{IN}=.002sin(400t)



$$v_{OUT} = A_V v_{IN}$$

$$v_{OUT} = -177 \bullet .002\sin(400t) = -0.354\sin(400t)$$

$$V_{OUT}(t) \cong V_{OUTQ} + A_V v_{IN}$$

$$V_{OUT} \cong 7.4V - 0.35 \bullet \sin(400t)$$

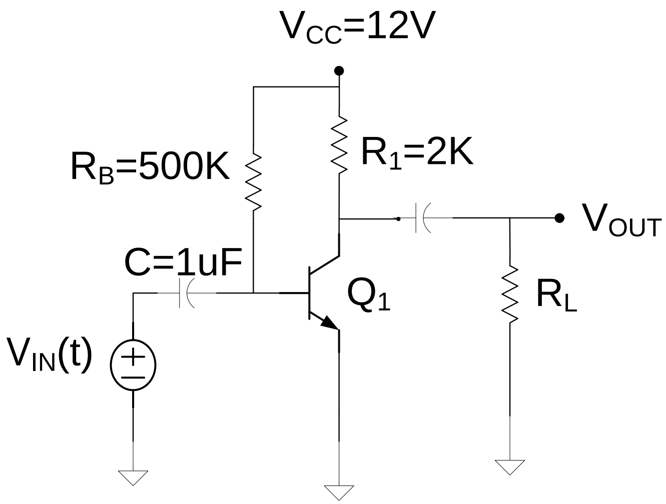
This example identified several useful characteristics of amplifiers but a more formal method of characterization is needed!

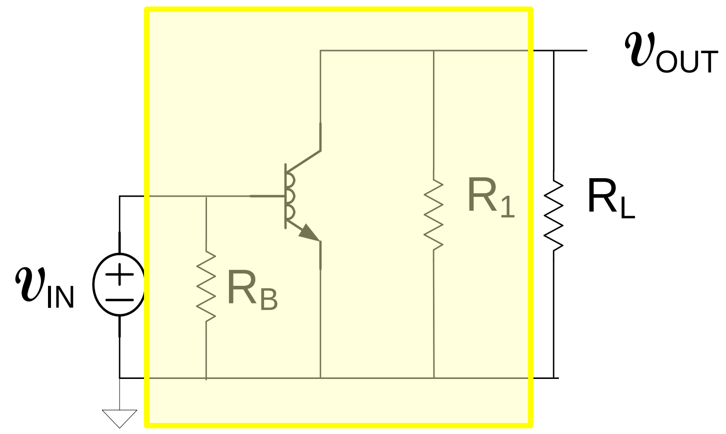
Amplifier Characterization

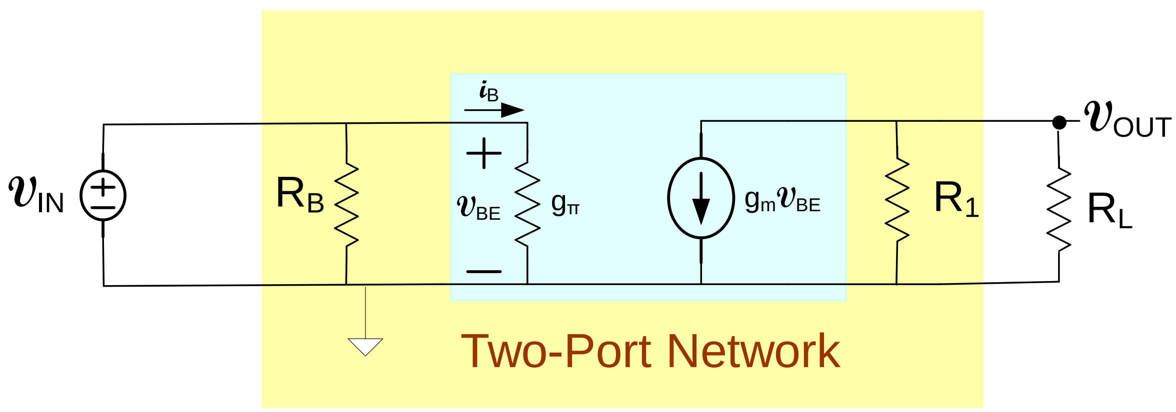
- Two-Port Models
- Amplifier Parameters

Will assume amplifiers have two ports, one termed an input port and the other termed an output port

Two-Port Representation of Amplifiers



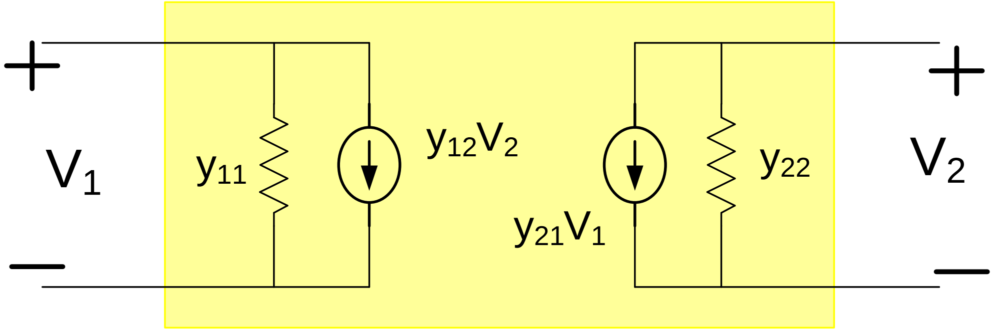




- Two-port model representation of amplifiers useful for insight into operation and analysis
- Internal components to the two-port can be quite complicated but equivalent two-port model is quite simple

Two-port representation of amplifiers

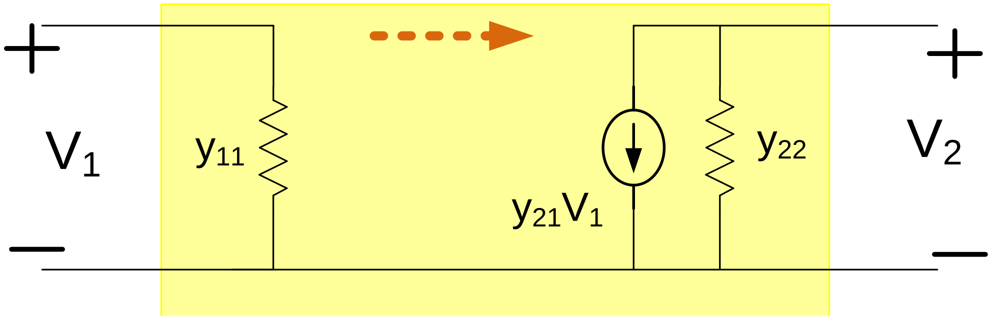
Amplifiers can be modeled as a two-port for small-signal operation



In terms of y-parameters

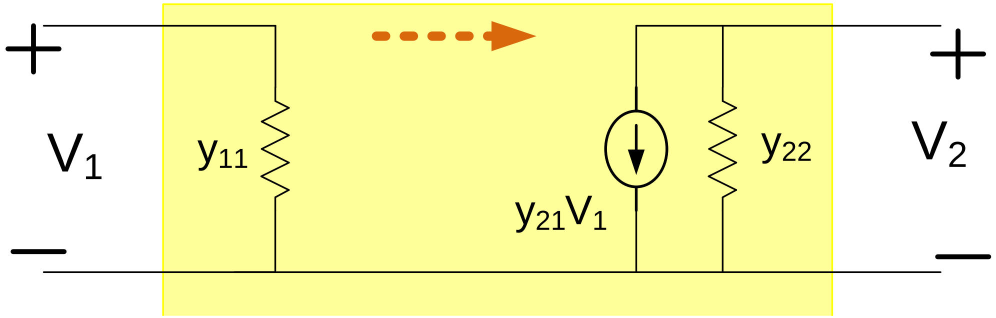
Other parameter sets could be used

- Amplifier often **unilateral** (signal propagates in only one direction: wlog y_{12}=0)
- One terminal is often common

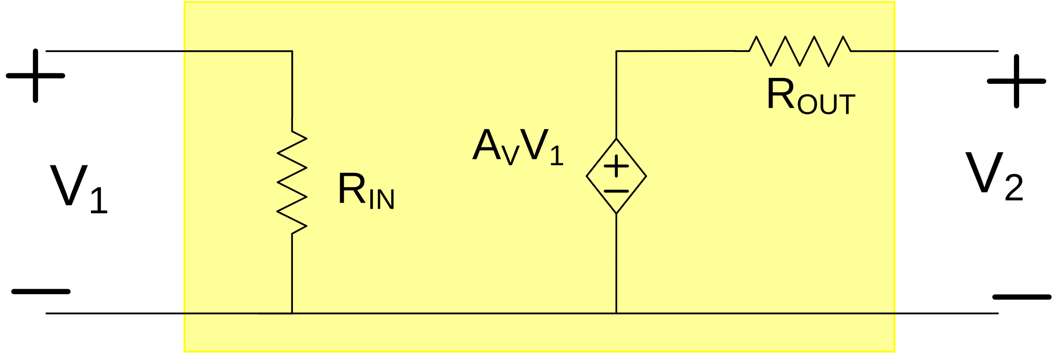


Two-port representation of amplifiers

Unilateral amplifiers:



- Thevenin equivalent output port often more standard
- R_{IN}, A_V, and R_{OUT} often used to characterize the two-port of amplifiers



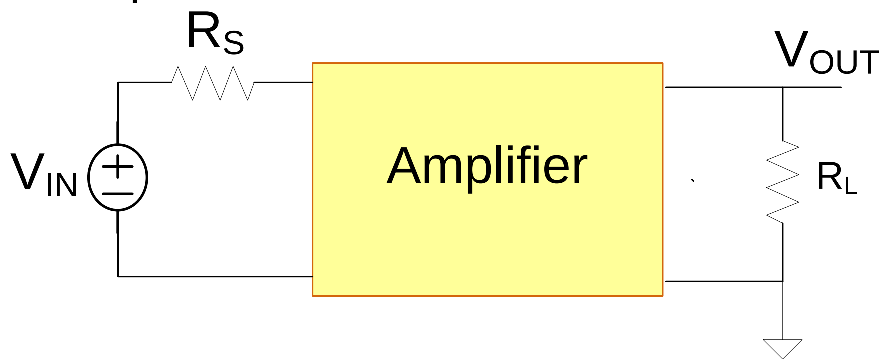
Unilateral amplifier in terms of “amplifier” parameters

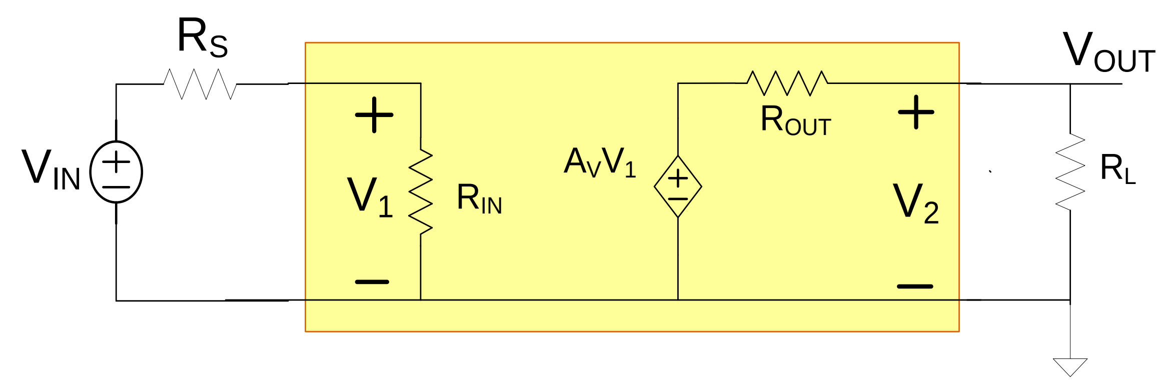
$$R_{IN} = \frac{1}{y_{11}} \qquad A_V = -\frac{y_{21}}{y_{22}} \qquad R_{OUT} = \frac{1}{y_{22}}$$

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 1: Assume amplifier is <u>unilateral</u>





$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT}}\right) A_V \left(\frac{R_{IN}}{R_S + R_{IN}}\right) V_{IN}$$

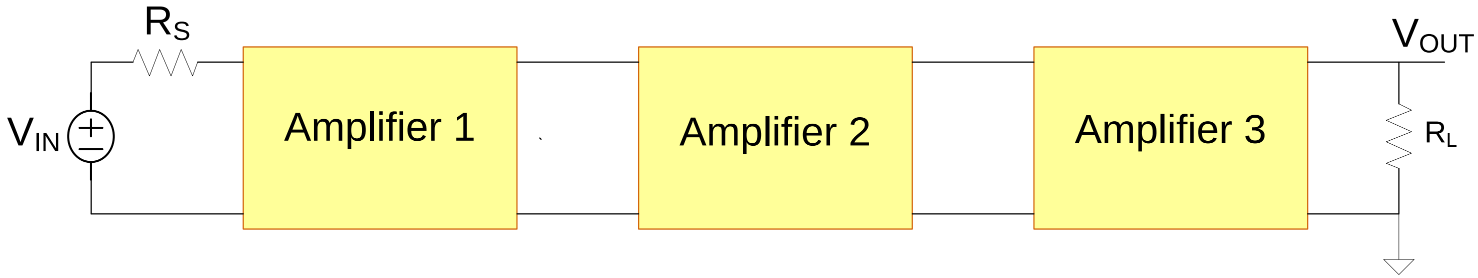
$$A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT}}\right)\left(\frac{R_{IN}}{R_S + R_{IN}}\right) A_V$$

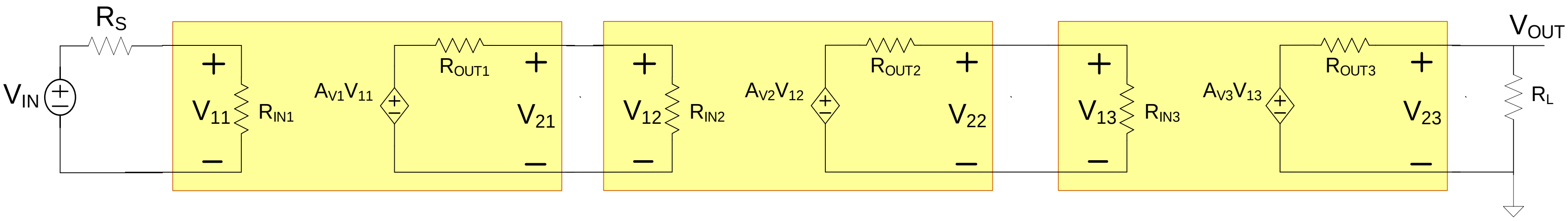
- Can get gain without reconsidering details about components internal to the Amplifier !!!
  - Analysis more involved when not unilateral

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 2: Assume amplifiers are unilateral





$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT3}}\right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}}\right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}}\right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}}\right) V_{IN}$$

$$A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT3}}\right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}}\right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}}\right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}}\right)$$

- Can get gain without reconsidering details about components internal to the Amplifier !!!
- Analysis more involved when not unilateral

Stay Safe and Stay Healthy !

End of Lecture 27